The Observer's Fallacy: Why Anomalous Behavior May Be Rational

Dissertation
for the Faculty of Economics, Business Administration and Information Technology of the University of Zurich

to achieve the title of
Doctor of Philosophy
in Management & Economics

presented by

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from Hohentannen TG

approved in April 2011 at the request of

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The Faculty of Economics, Business Administration and Information Technology of the University of Zurich hereby authorises the printing of this Doctoral Thesis, without thereby giving any opinion on the views contained therein.

Zurich, 6. April 2011

Chairman of the Doctoral Committee: Prof. Dr. Dieter Pfaff

To my parents.

Acknowledgments

During the last years I enjoyed much support and advice from various people and institutions.

I am especially grateful to my thesis advisors Ernst Fehr and Ulrich Kaiser. Ernst Fehr encouraged me to start this endeavor. His strong support and guidance made this PhD project possible in the first place. Many thanks go to my second advisor, Ulrich Kaiser, for his great support and efforts.

I owe my deepest gratitude to my friends and coworkers Helga Fehr-Duda and Adrian Bruhin. This thesis would not have been possible without the constant support of Helga and the inspiring discussions I had with her. Adrian, on the other hand, contributed much to my understanding of applied econometrics.

I am also indebted to the following persons and institutions for their support, their feedback or valuable discussions: Renate Schubert, María Sáez-Martí, the members of the Institute for Empirical Research in Economics at the University of Zurich, the Seminar in Statistics at the ETH Zurich, the organizers and participants of the Mannheim Empirical Research Summer School 2006, the organizers and participants of the Summer Institute in Behavioral Economics in Trento 2008, as well as the participants of the many conferences and seminars I attended and where I presented my research during the last years.

I gratefully acknowledge financial support of the Swiss National Science Foundation (grant 100012-109907 and 100014-120705) and the Swiss Federal Office of Energy.

Finally, I would like to thank my family for their everlasting support and motivation during my studies.

Contents

Chapter 1

Introduction

Most economic decisions, such as how much to save for retirement and whether to start a business, involve choices among options with delayed and uncertain consequences. Not surprisingly then, theories of choice over time and choice under risk constitute essential building blocks of a broad range of economic models. To be useful for policy makers and economists, these fundamental theories must capture the relevant aspects of decision making, and they must be able to produce sufficiently accurate predictions. The standard theories of intertemporal and risky choice, discounted utility theory (Samuelson, 1937) and expected utility theory (von Neumann and Morgenstern, 1947), seem not to meet these requirements. While they are often taken as criteria for rational choice, it is now well known that people tend to act in ways which systematically violate these theories' core axioms (Starmer (2000), Frederick et al. (2002)). In a nutshell, the literature documents the following anomalous behaviors. People are typically more impatient in the short run than in the long run, and they typically overweight small-probability outcomes and underweight large-probability outcomes. Their choice behavior seems to depend on both, outcome magnitude and outcome sign. Other puzzling findings are the vast behavioral differences among subjects from the same cohort, the considerable temporal instability of individual behavior, and the apparent situation-dependency of discount rates and risk attitudes. Taken together, these findings suggest that the standard theories only provide an incomplete characterization of effective choice behavior.

Many attempts have been made to solve this issue, the most prominent ones being hyperbolic discounting models (Ainslie (1975), Loewenstein and Prelec (1992), Laibson (1997), among others) and models incorporating nonlinear probability weights (Quiggin (1982), Tversky and Kahneman (1992), among others), but these "solutions" entail a number of novel hitches. The probably most critical ones are the following two.

First, in the literature these models are generally interpreted as pure preference models. That is, anomalous behavior is attributed in its entirety to the decision maker's preferences. From the perspective of standard economic theory, these preferences are exotic because they do no longer obey the standard theories' key axioms. In particular, hyperbolic discounting models introduce dynamically inconsistent time preferences, and, hence, violate stationarity, whereas models incorporating nonlinear probability weighting violate independence. Making such strict assumptions about preferences can pose severe problems. Crawford (1990) and Krusell and Smith (2003), for instance, point out that dropping the standard axioms can complicate game-theoretic analyses in a considerable way.[1] Moreover, by being pure descriptions of behavior, these models remain silent about the sources of anomalous behaviors. They (implicitly) assume that differences in subjects' behavior are exclusively due to differences in their preferences, but they neglect other important factors affecting choice behavior, such as constraints limiting the decision maker's scope of action. This lack is of particular practical relevance. Policy makers, for example, need to know what drives the behavior they observe, and they need to know whether this behavior is due to rational reasons or other factors. Only with a profound knowledge of where and how to intervene is it possible to implement policies which affect the right people in the right way. As the descriptive theories do not provide an answer to these questions, their potential applications remain quite limited. To sum up, useful theories of intertemporal and risky choice addressing this issue should retain the tractability and normative appeal of the standard theories, while, at the same time, they should improve their descriptive validity and provide a clear and intuitive story for systematic departures from standard behavior.

Second, these models do not provide a unifying approach to anomalous behavior in intertemporal and risky choice. None of the models proposed gives a coherent explanation for all the stylized facts of the domain they are concerned with. In fact, models such as hyperbolic discounting capture one single anomaly only, in this particular case the decline of discount rates in time horizon. They fail, however, at predicting the vast number of at least equally important facts, such as the dependency of discount rates on outcome magnitude and outcome sign. Some attempts were made to cope with a broader set of anomalies (e.g. Loewenstein and Prelec (1992) and Tversky and Kahneman (1992)), but

[1] Krusell and Smith (2003) discuss the problem of multiple equilibria in settings with competing "selves" for the case of quasi-geometric discounting. Crawford (1990) shows that the Nash equilibrium concept requires independence and proposes an extension to make analyses possible under quasi-concave preferences.

they do so by imposing an even larger number of ex-ante assumptions on preferences.[2] Why is it so important to have a coherent explanation at hand? The literature on the anomalies in intertemporal and risky choice suggests that all the stylized facts listed above usually appear at the same time and not in isolation. This raises the suspicion that all these findings have one common explanation. Only when the mechanism generating these patterns can be uncovered is it possible to also understand interactions between these anomalous behaviors, and, hence, their consequences. Moreover, choices over time and choices under risk are very similar in their nature and in the way people deviate from the standard models' predictions (Prelec and Loewenstein, 1991). It seems therefore likely that anomalous behavior in the two domains has the same cause or that this behavior, at least, carries over from one domain to the other. In this case, an integrated explanation for the anomalies should go much further than just explaining all patterns in one single domain, but, instead, it should embed both choice domains into one coherent framework. Such a unifying approach would allow economists to get a better understanding of the similarities and interdependencies between choices over time and choices under risk, an understanding of particular relevance since most real-world choices involve both, a time and a risk dimension.[3] All these points stress the ultimate need for theories which provide a clear and unifying explanation for the patterns observed.

This thesis addresses these issues. I present three papers which offer novel explanations for anomalously appearing behavior in choice over time and choice under risk, and provide the following key insights.

First, I show that environmental factors rather than exotic preferences may drive systematic departures from the standard theories' predictions. I consider two kinds of such factors: *Inherent uncertainty* which generates an additional layer of uncertainty over even allegedly guaranteed future outcomes, and *environmental constraints* which limit the decision maker's access to the commodity of interest.[4] As it turns out, these factors

[2]Loewenstein and Prelec (1992), for instance, capture magnitude-dependent time preferences by assuming that the value function has increasing elasticity, and they capture sign-dependent time preferences by assuming that the value function is more elastic for losses than for gains. With cumulative prospect theory, Tversky and Kahneman (1992) account for sign-dependency, but not magnitude-dependency (see Holt and Laury (2002) and Fehr-Duda et al. (2010) for empirical evidence on stake effects in choice under risk). They also impose restrictions on the value function.

[3]While there is always some uncertainty about the materialization of future outcomes (see first paper of this thesis), there are some examples for risky choices where time discounting plays no role. Among others, these are casino gambling and the situation subjects are confronted with in controlled risky choice experiments (see third paper of this thesis).

[4]It has long been recognized that other factors than pure preferences drive behavior. Most notably, the early work on intertemporal choice (Böhm-Bawerk (1921), Fisher (1930)) provides an extensive discussion on such drivers. Surprisingly, however, these factors received little attention during the last few decades

can lead even decision makers obeying the standard axioms on the level of preferences to reveal behavior consistent with the findings documented in the literature. Anomalous behavior then results from the discounted utility maximizer's or the expected utility maximizer's *rational* responses to her environment. Hence, this thesis does not only provide an intuitive explanation for systematic departures from standard predictions, but it also demonstrates that these findings can have rational reasons. Put differently, it is the observer who eventually wrongly ascribes anomalous behavior to irrational, exotic preferences rather than to potentially rational responses to environmental factors.

Second, as it turns out, my theory is able to explain all anomalies *at one single blow*. As all the stylized facts of behavior may originate from the decision maker's reaction to her limited access to the commodity of interest, there is no need to impose a plethora of ex-ante assumptions on preferences to capture these patterns. Since all anomalous behaviors can be explained within a theory retaining the standard assumptions about preferences, my approach builds the bridge between the standard preference models of intertemporal and risky choice and effectively observed behavior in these two domains. That is, it brings together the normative appeal and tractability of the standard models with the descriptive validity and intuition policy makers and economists ask for.

Third, taken together, the papers in this thesis develop a *unifying* approach to intertemporal and risky choice behavior. They do not only provide a coherent explanation for the anomalies in one single domain, but also work out a common explanation for departures in *both* domains. In principle, there are two different pathways over which the risk domain may interweave with the time domain. On the one hand, the same basic intuition can explain the apparent systematic departures from standard predictions in both choice domains. This is the case in the presence of environmental constraints. On the other hand, for decision makers prone to probability distortions, the core anomalies of intertemporal choice naturally arise as a consequence of inherent uncertainty. The research presented in this thesis therefore contributes to a better understanding of the similarities and interactions between the two choice domains, an understanding inevitable for most applications of these theories.

Finally, the findings presented here have a number of *important implications* relevant for both, policy makers and economists. First of all, it is shown that the most prominent anomalies of intertemporal and risky choice, hyperbolic discounting and probability distortions, are exhibited by the same people. This empirical result supports the conjecture that systematic departures from standard theories' predictions in both choice domains

of decision research.

have one common source. The two explanations I offer show how the environment can influence choice behavior and foster this result. My theories generate much more precise and testable predictions with respect to behavior in changing environments than most other theories of intertemporal and risky choice. This is the case since behavior is explicitly modeled as an outcome of the interplay between the decision maker's preferences and her environment. Furthermore, I argue that conditions where the environment is subject to change should make it possible to separate rational from less rational motives for anomalous behavior. Some of these less rational intuitions are presented in the course of this thesis. In particular, these are self-imposed constraints, optimistically biased beliefs or evolutionary-shaped preferences. Disentangling rational from less rational reasons is of particular relevance for developing suitable policy instruments which help economic agents to behave in a more rational way without harming those that already do (Camerer et al., 2003).[5] Policy interventions based on wrong assumptions about preferences or rationality may give economic agents an incentive to depart from optimal behavior, and, hence, may - in the worst case - induce severe welfare losses. The theories proposed here make clear that the decision maker's environment provides a natural starting point for policy interventions. Typically, such policies will aim at improving the surrounding conditions the decision maker is confronted with. For example, in the presence of institutional uncertainty general norms of *pacta sunt servanda* and the possibility to ensure against background risks can promote retirement savings. Similarly, when environmental constraints limit the economic agents' scope of action, facilitating economic agents' market access can help them overcome these limitations.

The remainder of this paper is structured as follows. Chapter 2.1 to 2.3 give a summary of the main results of the three papers in this thesis.

A first paper, introduced in Chapter 2.1, deals with the most prominent anomalies in choice over time and choice under risk, hyperbolic discounting and probability distortions, and shows that the same people are prone to these systematic departures from the standard predictions. It argues that inherent uncertainty together with probability distortions can rationalize hyperbolic discounting.

A second paper, introduced in Chapter 2.2, shows that the apparent anomalies in intertemporal choice naturally arise for rational discounted utility maximizers if they face liquidity constraints and hold positive income expectations. I find considerable support for this theory in two experimental data sets.

A third paper, introduced in Chapter 2.3, closes the gap between the other two theo-

[5] Under the restriction that there are no negative external effects caused by rational behavior.

ries. It shows that the same basic argument motivating anomalies in intertemporal choice can also be applied to risky choice. Anomalous behaviors can emerge for expected utility maximizers facing environmental constraints. Together with one of the other two theories, this theory can provide a unifying explanation for the anomalies in intertemporal *and* risky choice.

Chapter 3 concludes and gives a short outlook to my future research agenda.

Chapter 2

Contents of this Thesis

2.1 Viewing the Future through a Warped Lens: Why Uncertainty Generates Hyperbolic Discounting

Experimental evidence suggests that, on aggregate, behavior in choice over time and choice under risk is best characterized by hyperbolic discounting and probability distortions. Most recent descriptive theories in these two domains capture these findings. Despite the significant parallels between intertemporal and risky choice (Prelec and Loewenstein, 1991), little research has been done to examine the link between these two domains empirically. Most surprisingly, there seems to be no previous study examining whether these anomalous behaviors are exhibited by the same people. An answer to that question, however, is of central interest from both a theoretical and an empirical point of view. Only if there is a systematic relationship between departures from standard predictions in the two domains, a theory which ascribes these departures to the same factor would be legitimate. In addition, predicting behavior for situations where both, time and risk, play a role requires the researcher to understand the alleged interactions between these two domains.

The first article of this thesis explores the link between hyperbolic discounting and probability distortions on the individual level and gives an interpretation for the results found in the empirical data.

Together with my coauthors, Helga Fehr-Duda and Adrian Bruhin, I present the following insights. Based on an experiment with salient monetary incentives we demonstrate that there is a strong and significant relationship between the degree of decreasing discount rates and greater departures from linear probability weighting at the level of individual behavior. People departing more strongly from the standard model's prediction in one domain also depart more strongly from the standard model's prediction in the other domain. Other factors such as the curvature of the utility function or socioeconomic characteristics do not contribute to explaining this link. Our results suggest that anomalous behaviors in both choice domains have a common source.

A possible interpretation for these findings is provided in the second part of the paper. It is argued that the relationship between hyperbolic discounting and probability distortions can be rationalized by the uncertainty inherent in any future event. Even allegedly guaranteed future outcomes are uncertain by their very nature as unforeseen circumstances may prevent these outcomes from materializing. For example, sudden illness may make it impossible for the decision maker to collect a promised reward. In the presence of such uncertainty, the evaluation of future outcomes crucially depends on the decision maker's risk preferences. While, in this analysis, we stay agnostic about the reasons for probability distortions, our framework posits a possible causal link between the anomalies in the two domains. In particular, decreasing discount rates may be generated by people's proneness to probability distortions. Put differently, hyperbolic discounting is driven by viewing the uncertain future through a warped lens, produced by systematic distortions of probabilities. The theoretical framework we derive not only organizes our experimental findings but also accounts for previous evidence of interactions of time and risk.

The findings of this study have a number of important implications. First, while theories of choice over time and choice under risk evolved to a large extent in two separate branches, our results make clear that we should head towards a coherent explanation for anomalies in both domains. This is of particular importance since the vast majority of choices we are confronted with in the real world involve both a time and a risk dimension. An integrated approach can therefore contribute to a better understanding of the interactions between these two choice domains and, hence, may be inevitable when predicting outcomes in more complex settings.

Second, if inherent uncertainty is the driving factor of anomalous behavior, it is possible to derive a number of policy recommendations which help economic agents to temporally allocate quantities of the commodity in question in agreement with their preferences. According to our framework, the link between the risk and the time dimension becomes only effective if there is uncertainty about the materialization of future outcomes. As a result, policies reducing this inherent uncertainty may attenuate the ties between the two domains, and, hence, may reduce excessive short-run impatience.

The framework proposed in this article takes probability distortions as given. While it rationalizes the link between the time and the risk domain, it does not make any statements about the rationality of these probability distortions per se. There may be many reasons for weighting probabilities nonlinearly. I provide a rational explanation for it in the third article of this thesis. There, I argue that apparent probability distortions can be the result of an expected utility maximizer's rational response to constraints imposed by her environment.[1]

An alternative pathway to the relationship between intertemporal and risky choice behavior is presented by the following two articles. They demonstrate that environmental constraints can produce the apparent anomalies in both choice domains, and, hence, that such constraints can lead to some very similar conclusions as the explanation based on inherent uncertainty.

2.2 Rational Planners or Myopic Fools? Liquidity Constraints, Positive Expectations and Anomalies in Intertemporal Choice

Empirical research on intertemporal choice documents a broad number of anomalies other than the decline of discount rates in time horizon. Observed discount rates also tend to decline in outcome magnitude, and seem to be larger for gains than for losses. These systematic departures from discounted utility theory's predictions usually appear at the same time. There are a number of additional puzzling patterns, such as the vast heterogeneity in behavior, its apparent temporal instability, and its ostensible dependency on situational factors, which are hardly explainable within traditional preference theories. This poses the question of whether there is a unifying approach explaining all these anomalies, and, if yes, whether these patterns can be explained within rational choice and without making exotic assumptions about the decision maker's risk preferences.

The second article of this thesis answers these questions. It presents a theory which explains anomalous behavior within the bounds of the standard model of intertemporal choice, and it provides empirical support for this explanation. I show the following results.

[1] As can be shown, under this particular kind of probability distortions the "Warped Lens"-framework can also explain anomalies in intertemporal choice beyond hyperbolic discounting, such as the dependency of discount rates on outcome magnitude and outcome sign. An elaborate discussion of this interaction is beyond the scope of this thesis.

First, there are situations where it is optimal for discounted utility maximizers to exhibit a behavior consistent with the apparent anomalies of intertemporal choice. Liquidity constraints can prevent rational discounters to temporally (re-)allocate future consumption in a way they prefer to. If they have an aversion towards consumption fluctuations and rationally expect their income to substantially rise in the future, such constraints can force them to allocate new cash inflows at earlier rather than later dates. As a result, the discount rate they reveal may exceed their rate of time preference. Observed discounting behavior then mimics that of a hyperbolic discounter with magnitude- and sign-dependent preferences. Probably most interestingly, it can be shown that if an economic agent holds rational expectations, hyperbolic discounting can be dynamically consistent. My approach can also account for other phenomena which have remained largely unexplained so far, such as increasing discount rates, the vast heterogeneity, the ostensibly dynamic instability of behavior, and situation-specific discounting behavior.

Second, I give an alternative, boundedly rational explanation for these results. Anomalous looking behavior is not necessarily caused by *rational planner*, but may be due to *myopic fools*, i.e. decision makers with standard preferences but optimistically biased expectations. For the latter group, anomalously appearing behavior emerges even if they have unlimited access to liquidity. Put differently, the optimism bias drives a wedge between the present and the future. The resulting model shares some characteristics with quasi-hyperbolic discounting (Laibson, 1997), but is able to capture a much wider spectrum of anomalies.

Third, I use data from two experiments to test some of my theory's most central hypotheses. The two experiments involve the same choices, but were conducted with two different groups of college students which are likely to hold different income expectations. I find strong support for my approach on the descriptive level for both groups. Income expectations seem to explain a large part of the anomalous patterns found in the data. Moreover, estimation of a structural model indicates that the proposed model performs much better than hyperbolic discounting. The group of students entering the job market sooner indeed reveal higher consumption expectations than the reference group. The theory is therefore able to explain behavioral differences by differences in the environment these groups face.

These findings have a number of strong implications. First, since very similar behavior may be caused by rational planners and myopic fools, policy makers must be careful in employing interventions which aim at helping economic agents to behave in a more rational way without harming those that already do (Camerer et al., 2003). Such policies may give rational economic agents an incentive to depart from optimal behavior. Policy makers may therefore search for mechanisms which distinguish between different sources of anomalous behavior. Dynamic consistency may be one criterion which can serve as a starting point for developing such mechanisms.

Second, the proposed model helps to understand the effect the environment has on choice behavior. It makes precise and testable predictions for given environments and given preferences, and may provide an explanation for the vast heterogeneity in discounting behavior and its dependence on situational factors.

The theory presented in this article only covers anomalies in choice over time, however. This raises the question whether similar reasoning can rationalize systematic departures from standard predictions in the domain of choice under risk. The following article answers this question, and lies the foundation for a unifying approach to anomalous behavior in both domains.

2.3 Preferences or Constraints? A Rational Explanation for Probability-Dependent Risk Attitudes

On aggregate, people violate expected utility theory in similar ways as they violate discounted utility theory (Prelec and Loewenstein, 1991). Risk taking behavior appears to depend on probabilities, outcome magnitude and outcome sign. Congruent with the domain of choice over time, risk taking behavior is very heterogenous and temporally unstable, and it seems to depend on situational factors. This suggests that a similar argument may explain anomalous behaviors in choice over time and choice under risk.

To provide such an explanation for the domain of choice under risk is the goal pursued by the third article of this thesis. In this article, I argue that environmental constraints similar to those described in the second article can explain a broad number of expected utility violations. The following results arise.

First, I show that anomalous behavior is not necessarily driven by the decision maker's probabilistic risk attitudes, i.e. her preferences, but may result naturally from the rational expected utility maximizer's response to exogenous constraints, i.e. her environment. The decision maker I consider obeys the standard axioms, but she has to bear some negative consequences in form of costs if the terminal outcome she achieves does not allow her to attain a certain minimal subsistence level. For example, a consumer has to meet her existential needs, has to honor her contracts and has to settle her bills. If she does not, she gets prosecuted or, in the extreme case, will starve. If this is not possible with any option in the choice set and there is no opportunity to overcome this limitation by accessing a market, she is constrained. As I show, these environmental constraints can force her to take higher risks than her preferences would suggest.

Second, like in the case of intertemporal choice, I give an alternative explanation for anomalous behavior. Similar behavior may also emerge if constraints are self-imposed, i.e. if the minimal subsistence level is indeed an aspiration level. Such behavior is hard to defend as rational and its sources are difficult to verify. Alternatively, risk attitudes may also be an innate characteristic of preferences which adapted over thousands of years of human evolution. In this sense, the theory I propose may provide the underlying mechanism which shaped these preferences.

The implications of these findings are manifold. First, it is, to the best of my knowledge, the first theory which provides a rationalization of probability distortions.[2] This result challenges the view that rational economic agents should reveal linear probability weighting. Put differently, it calls into question that linear probability weighting is the benchmark for rational choice. Moreover, as pointed out earlier, there are two pathways through which the model proposed in this article can provide a coherent and rational explanation for the anomalies in choice over time and choice under risk.

Second, the policy implications are very similar to that proposed in the second paper. The same interventions, in particular policies which facilitate market access, may help economic agents irrespective of whether they face intertemporal or risky choice problems or a combination of both.

[2]Some economists, however, argue that emotions drive departures from standard predictions and that anomalous behavior due to the anticipation of these emotions can be considered as rational (e.g. Loomes and Sugden (1982) and Loomes and Sugden (1986)). I discuss the weaknesses of such approaches in the literature section of the paper.

Chapter 3

Conclusion

Descriptive theories proposed in response to the anomalies in intertemporal and risky choice follow quite a radical route. They capture anomalous behavior by preferences alone. The fast adoption of these theories is probably the reason why many researchers forgot about other important drivers of behavior. However, this is by far not the only novel problem these theories bring with. As, in these theories, anomalous behavior arises by assumption and not endogenously, every additional anomaly they capture also requires additional restrictions on preferences. Hence, these theories do not provide a unifying approach to the anomalies in the domain they deal with. A direct consequence of modeling behavior the way they do is that these theories drop many desirable properties of the standard theories. As a result, they may hamper economic analyses in fundamental ways. Moreover, as these theories of choice over time and choice under risk developed in separate branches, they do not provide an integrated approach to the two domains, and, hence, remain silent about how behavior between these two domains interacts.

This thesis presents a solution for these issues and shows that they can be resolved elegantly by explicitly taking into account the environment the decision maker is confronted with. This leads to a number of novel insights. For example, it suggests that, although many researchers have accepted constant discount rates and linear probability weights as benchmarks of rational choice, this presumption may be fundamentally wrong. There are rational reasons for departing from discounted utility theory's and expected utility theory's standard predictions. This is of particular relevance for policy makers interested in employing paternalistic regulations, i.e. regulations which help on the individual level and encourage people to act in a more rational way. This thesis discusses potential instruments which policy makers can implement to foster behaviors in accordance to agents' preferences.

Another focal point is that the approach presented in this thesis retains the desirable properties of the standard theories on the level of preferences. The theories presented can therefore simplify game-theoretic analyses in considerable ways when compared to theories incorporating hyperbolic time preferences or nonlinear probability weighting. As can be shown, for rational decision makers, they permit the researcher to conduct these analyses without messing around with multiple equilibria problems or alternative equilibrium concepts.

All these features are important for most applications of such theories. For economists theories of choice over time and choice under risk constitute essential building blocks of a broad range of economic models. They should therefore be both, descriptively valid and normatively appealing. Moreover, most management decisions rely on assumptions about preferences of second or third parties. Managers must therefore know what effectively drives behavior.

The present thesis opens up a whole branch of research. I see three principal directions worth pursuing for future research.

First, there is need for causal tests of the theories proposed. In this thesis I am interested in rationalizing observed discount rates and risk-taking behavior by controlling for environmental factors. Controlled laboratory experiments, however, are easily carried out and may complement the findings presented here. They can help to get a better understanding of the prevalent drivers of behavior.

Second, for the policy maker it is important to have mechanisms at hand which distinguish between different motives underlying behavior. Only with this knowledge is it possible to decide whether and how to intervene. In this thesis it is argued that anomalously appearing behavior may be generated by both, fully rational economic agents and boundedly rational economic agents. Possible reasons for being boundedly rational are that economic agents face self-imposed constraints, hold biased beliefs or have evolution-adapted preferences. In the course of this thesis possible starting points for the development of suitable mechanisms are proposed.

Finally, the basic ideas pursued in this thesis may also be applied to other choice domains. Environmental constraints, for example, can help to understand the evolutionary foundations of social preferences. In this sense, the case of risk taking behavior is just one of possibly many examples in which apparent violations of classical economic theory may be traced back to constraints rather than to a failure of the standard preference model.

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Appendix A

Viewing the Future through a Warped Lens: Why Uncertainty Generates Hyperbolic Discounting

This chapter is joint work with Helga Fehr-Duda and Adrian Bruhin.

Another version of this chapter has been published as *IEW Working Paper No. 510* and is currently under review at *Journal of Risk and Uncertainty.*

"Future income is always subject to some uncertainty, and this uncertainty must naturally have an influence on the rate of time preference, or degree of impatience, of its possessor."

Fisher (1930)

A.1 Introduction

It has long been recognized by practitioners and theorists alike that the domains of choice under risk and over time are intimately related. Unlike gambling in the casino, where uncertainty is resolved almost immediately, many real world decisions, and probably the most important ones for our happiness, involve risky consequences materializing over the course of time. In the realm of economic theory, the dimensions of risk and time are treated as largely independent attributes, modeled in an equivalent way (Prelec and Loewenstein, 1991): The classical models of choice, expected utility theory (EUT) and discounted utility theory (DUT), view decision makers as maximizing a weighted sum of utilities with the weights representing either probabilities or discount factors, respectively.

A large body of empirical evidence has challenged the validity of EUT and DUT as descriptive models of choice, however. In the domain of risk, one of the best documented phenomena concerns the *Common Ratio Effect*: Often, people's preference for a smaller more probable outcome over a larger less probable one changes in favor of the larger outcome when both outcome probabilities are scaled down by a common factor. This pattern of behavior constitutes a violation of the independence axiom of EUT (Kahneman and Tversky, 1979; Starmer and Sugden, 1989).[1] The stationarity axiom of DUT, according to which preferences should depend on the absolute time interval between delivery of the objects, has met a similar fate. The *Common Difference Effect* describes the empirical regularity that preference for a smaller earlier payoff over a larger later payoff reverses when both payoffs are shifted into the more remote future, keeping the timing difference constant (Thaler, 1981; Benzion et al., 1989).

Researchers have reacted to these anomalies by relaxing the assumptions on the corresponding decision weights while leaving the overall structure of the models intact. Violations of independence can be captured by a suitable nonlinear transformation of the probabilities, as discussed by Quiggin (1982) and Tversky and Kahneman (1992). Violations of stationarity, on the other hand, are accounted for by allowing the discount factors

[1] Prominent special cases are the Allais paradox (Allais, 1953) and the Bergen paradox (Hagen, 1972).

to decline hyperbolically in time, i.e. at a decreasing rate (Ainslie, 1991; Laibson, 1997; Prelec, 2004). These generalizations seem to perform much better at explaining aggregate choices than do EUT and DUT (Rachlin et al., 1991; Harless and Camerer, 1994; Hey and Orme, 1994; Myerson and Green, 1995; Kirby, 1997). At the individual level, however, there is vast heterogeneity in observed behavior in both decision domains (Hey and Orme, 1994; Chesson and Viscusi, 2000; Abdellaoui et al., 2010a; Bruhin et al., 2010) and it is an open question whether the superior fit of the generalized models is a manifestation of common regularities of individual behavior. Clearly, endeavors of integrating risk taking and intertemporal choice in one single model, such as for example in Prelec and Loewenstein (1991), only make sense if the latter is the case. The question then arises whether violations of independence and stationarity are actually committed by the same people.

In the empirical literature, the relationship between individuals' attitudes towards risk and delay has been examined from various different angles. One strand of the literature focuses on people's risk tolerance measured independently from their degree of impatience. These studies find that more risk averse people tend to discount the future more heavily (Leigh, 1986; Anderhub et al., 2001; Eckel et al., 2004). Discount rates are inferred directly from choices over dated monetary amounts and, therefore, their measurement is confounded by the curvature of the utility function. Andersen et al. (2008) correct for utility curvature and still find a positive, but much reduced, correlation in their predicted degrees of risk aversion and impatience. None of the studies so far have accounted for probability weighting and, therefore, they cannot address the question of whether departures from linear probability weighting are systematically related to departures from constant discounting. Similarly, the psychological literature has dealt with comparisons of highly reduced forms of discounting functions for delay and probability, ignoring utility curvature as well as probability weighting (Myerson et al., 2003). These findings also indicate moderate positive correlations between both types of discounting.

Another strand of the literature investigates people's choices when both risk and delay are present (Keren and Roelofsma, 1995; Ahlbrecht and Weber, 1997; Weber and Chapman, 2005; Noussair and Wu, 2006; Anderson and Stafford, 2009; Baucells and Heukamp, 2010; Coble and Lusk, 2010). These studies generally conclude that there are interaction effects between time and risk, such as risk tolerance increasing with delay, which are not easily justifiable within the frameworks of EUT and DUT. Again, probability weighting does not feature in any of these papers. A notable exception is the contribution by Abdellaoui et al. (2010b) who estimate individual probability weights over varying delays, but do not elicit discount functions for guaranteed payoffs.

Finally, some recent papers examine the effects of risk in the payment date, rather than in outcome magnitude. Parallel to the findings on delayed guaranteed outcomes, Chesson and Viscusi (2000) report discount rates to decline with time horizon. Moreover, Chesson and Viscusi (2003) show that aversion to timing risk is positively related to ambiguity aversion, suggesting that uncertainty may be processed similarly in both the dimensions of time and risk. In a follow-up study Onay and Öncüler (2007) argue that the prevalence of timing risk aversion, which runs against the predictions of EUT, can be accommodated within a rank-dependent model involving probability weighting. They did not test their conjecture empirically, however.

This brief review of the literature shows that, to the best of our knowledge, there is no previous study that investigates the same individuals' probability weights and discount functions. While evidence of hyperbolic discounting is occasionally reported, simultaneous estimates of individual probability weights are usually not provided. This lack may be due to the fact that a comparatively rich data set, and for that matter also a fairly sophisticated estimation strategy, is needed to be able to disentangle utility curvature and probability weighting.

In order to close this gap, we conducted an experiment with salient monetary incentives, which exhibits a number of distinguishing features: First, the experiment generated data rich enough to be able to estimate *individual* probability weighting functions and relate them to the same subjects' revealed discount rates. Second, in contrast to many previous discounting experiments, every single subject got paid for her intertemporal choices, involving substantial payoffs, in an incentive compatible manner. Third, we kept transaction costs equal across different payment dates in order to preclude confounding effects. Finally, we controlled for utility curvature.

We present the following experimental results. First, we show that subjects' departures from linear probability weighting are highly significantly associated with the strength of decreasing discount rates. The curvature of the utility function, however, seems not to be directly related to their decline. Second, estimation results are robust to controlling for socioeconomic characteristics, such as gender, age, experience with investment decisions and cognitive abilities. In fact, the only variable associated with decreasing discount rates turns out to be the degree of nonlinearity of probability weights, which explains a, by any standard, large percentage of the variation in the extent of the decline. In particular, cognitive abilities, as measured by the *Cognitive Reflection Test* (Frederick, 2005), cannot account for the link between proneness to probability distortions and hyperbolic discounting. Moreover, all our results are insensitive to model specification.

Our findings demonstrate that it is the same people who tend to violate the axioms of independence and stationarity. We suggest that this relationship is driven by a natural link between the domains of time and risk (Halevy, 2008; Walther, 2010): Arguably, only immediate consequences can be totally certain whereas delayed ones are uncertain by their very nature. For instance, a promised reward may, due to unforeseen circumstances, materialize later or turn out to be smaller than expected, or sudden illness or death may keep the decision maker from collecting her reward. For these reasons, future consequences are inextricably associated with uncertainty, implying that the decision maker's valuation of delayed outcomes not only depends on her *pure* time preference, i.e. her preference for immediate utility over delayed utility, but also on her perception of uncertainty and, consequently, on her risk preferences. We show theoretically that stronger departures from linear probability weighting entail more strongly declining discount rates, providing us with a theoretical underpinning of our experimental results. Figuratively speaking, hyperbolic discounting is driven by viewing the uncertain future through a warped lens, produced by systematic distortions of probabilities. This theoretical framework not only organizes our experimental findings but also accounts for previous evidence of interactions of time and risk. A number of studies detected preference reversals when either risk is added to temporal prospects (Baucells and Heukamp, 2010) or delay is added to simple risky prospects (Keren and Roelofsma, 1995; Weber and Chapman, 2005).

Our analysis suggests that institutionally generated uncertainties, such as lack of contract enforcement and weak property rights, may induce extreme short-run impatience even if people's pure rate of time preference is constant and relatively low. This insight is important because it implies that revealed behavior may be predominantly driven by environmental factors rather than by the underlying preferences themselves and, consequently, may be amenable to economic policy. While uncertainty may be an important channel through which hyperbolicity of discount rates is generated there may be other sources of hyperbolic discounting behavior as well. For instance, pure time preferences may be hyperbolic *per se*, as could be argued for addictive behavior. And when visceral motives, such as hunger or lust, come into play, uncertainty may not be the dominant dimension decision makers are concerned about. An excessive preference for the present may then be driven by factors other than potential disappearance of the object of desire.

The remainder of the paper is structured as follows: In Section A.2 we describe the experimental design and procedures. Section A.3 outlines our approach to estimation. Section A.4 presents our results. Section A.5 discusses our hypothesis on the role of risk preferences in time discounting. Section C.5 concludes.

A.2 Experimental Design

The experiment took place at the Institute for Empirical Research in Economics (IERE), University of Zurich, in May 2006. Participants were recruited from the IERE subject pool, which consists of students from all fields offered at the University of Zurich and the Swiss Federal Institute of Technology Zurich. In total, we analyzed 112 subject's responses.[2] The experiment consisted of two main parts, one dedicated to eliciting certainty equivalents for non-delayed risky prospects,[3] the other one to eliciting future equivalents and their corresponding imputed discount rates for temporal prospects involving guaranteed payments.[4]

We used similar procedures to elicit certainty equivalents and discount rates, in order to economize on subjects' cognitive effort. For both types of tasks we implemented choice menus containing a list of 20 varying alternatives which had to be judged against a fixed option. To familiarize subjects with the nature of the procedure, the instructions contained examples and trial problems. Besides a show up fee of CHF 10 (CHF 1 $\approx$ USD 0.8 at the time of the experiment), each subject was paid according to one of her risky choices and one of her temporal choices selected randomly at the very end of the experiment. Subjects received their compensation for the risky choices and the show-up fee in cash immediately after completion of all the tasks. The compensation for their intertemporal choices was paid out to them at the respective dates when they cashed in vouchers issued to them at the end of the experiment. Payment modalities are described in detail below. Subjects could work at their own speed. On average, it took them 1.25 hours to complete the experiment, including a socioeconomic questionnaire presented after the choice tasks.

A.2.1 Elicitation of Certainty Equivalents

Since the objective of the risk task was to obtain data on the basis of which individual probability weights could be estimated, a fairly large number of observations per person

[2]We omitted six subjects' responses from our analysis. Four subjects reported that they would not be able to cash in their delayed payments at the respective payment dates. Three of them would have been on vacation then, the fourth person had planned a long visit abroad. Hence, their choices were not informative of their time preferences. Concerning the remaining two subjects we could not disentangle utility effects from probability weighting effects. Nonetheless, our results do not change when we include these two individuals in the data set.

[3]The risk data was also used in Bruhin et al. (2010).

[4]Instructions are available upon request. The experiment was programmed and conducted with the software z-Tree (Fischbacher, 2007).

was needed. To elicit individual lottery evaluations, subjects were presented with 20 choice menus, each one involving a specific binary lottery $\mathcal{L} = (x_1, p; x_2)$, with $x_1 > x_2 \geq 0$, labeled *Option A* in Figure A.1. *Option B* in the choice menu represented the guaranteed alternatives, ranging from the higher lottery outcome x_1 to the lower outcome x_2. Every subject had to choose her preferred option in each row of the choice menu. In Figure A.1, a hypothetical subject prefers all guaranteed payments larger than CHF 36 to the stated lottery, and prefers the lottery in the remaining rows. This subject's valuation of the lottery, her certainty equivalent *ce*, is calculated as the arithmetic mean of the two amounts next to her indifference point, amounting to CHF 37 in the example here. The set of lotteries, listed in Table A.1, included a wide range of outcomes and probabilities. Every subject was confronted with this set of lotteries once. The choice menus appeared in an individualized random order.

Figure A.1: Choice Menu — Risk

	Option A	**Your Choice**		**Option B** *(guaranteed reward)*
1	Gain of CHF 50 with a probability of 75% *and* Gain of CHF 10 with a probability of 25%	A ○	● B	CHF 50
2		A ○	● B	CHF 48
3		A ○	● B	CHF 46
4		A ○	● B	CHF 44
5		A ○	● B	CHF 42
6		A ○	● B	CHF 40
7		A ○	● B	CHF 38
8		A ●	○ B	CHF 36
9		A ●	○ B	CHF 34
10		A ●	○ B	CHF 32
11		A ●	○ B	CHF 30
12		A ●	○ B	CHF 28
13		A ●	○ B	CHF 26
14		A ●	○ B	CHF 24
15		A ●	○ B	CHF 22
16		A ●	○ B	CHF 20
17		A ●	○ B	CHF 18
18		A ●	○ B	CHF 16
19		A ●	○ B	CHF 14
20		A ●	○ B	CHF 12

At the end of the experiment, after the subject had completed all the tasks, one row of one choice menu was randomly selected for payment. If the subject had opted for the lottery there, her decision was played out for real. If the subject had opted for the

Table A.1: Risky Prospects

p	x_1	x_2	p	x_1	x_2
0.1	20	10	0.25	50	20
0.5	20	10	0.5	50	20
0.9	20	10	0.75	50	20
0.05	40	10	0.95	50	20
0.25	40	10	0.05	150	50
0.5	40	10	0.5	10	0
0.75	40	10	0.5	20	0
0.95	40	10	0.05	40	0
0.05	50	20	0.95	50	0
0.1	150	0	0.25	40	0

Outcomes x_1 and x_2 are stated in CHF, p denotes the probability of x_1 materializing.

guaranteed payoff, the respective amount was paid out to her. On average, subjects earned CHF 37.22 in cash for the risk task, including the show-up fee of CHF 10, to be paid out immediately. Cash payments for the risk task were considerably higher than the local student assistant's hourly wage.

A.2.2 Elicitation of Discount Rates

Using a similar format as in the risk task we elicited individual discount rates for temporal prospects $\mathcal{T} = (x, t)$, with $x > 0$, over payments x delayed by t months. The choice menus, designed as in Figure A.2, contained 20 binary choices each.[5] Subjects had to choose between a guaranteed payment pe of CHF 60 the next day (*Option A*) and a guaranteed later payment x (*Option B*), delayed by two months or four months, respectively. The varying alternatives x were sorted in descending order from the highest amount to the lowest amount, incorporating an interest payment at a simple annualized rate of $\delta_t \in [0\%, 95\%]$ over the corresponding time interval $[0, t]$. These rates were exhibited in the right-most column of the choice menu.[6] The present amount of CHF 60 and the range of interest rates were chosen to provide salient incentives, so that deferring the reward was actually worthwhile. The arithmetic mean of the two monetary amounts next to the indifference point on the choice menu provided the imputed discount rate δ_t. The hypothetical subject in Figure A.2, for instance, is indifferent between CHF 60 and CHF

[5] A similar design was proposed by Coller and Williams (1999).

[6] The discounting experiment consisted of one additional task not reported here.

70.50, implying a discount rate of 52.5% *per annum*.

Figure A.2: Choice Menu — Time

	Option A *payment tomorrow*	**Your Choice**		**Option B** *payment in 4 months + 1 day*	
1		A ○	● B	CHF 79	95%
2		A ○	● B	CHF 78	90%
3		A ○	● B	CHF 77	85%
4		A ○	● B	CHF 76	80%
5		A ○	● B	CHF 75	75%
6		A ○	● B	CHF 74	70%
7		A ○	● B	CHF 73	65%
8		A ○	● B	CHF 72	60%
9		A ○	● B	CHF 71	55%
10	Amount	A ●	○ B	CHF 70	50%
11	of CHF 60	A ●	○ B	CHF 69	45%
12		A ●	○ B	CHF 68	40%
13		A ●	○ B	CHF 67	35%
14		A ●	○ B	CHF 66	30%
15		A ●	○ B	CHF 65	25%
16		A ●	○ B	CHF 64	20%
17		A ●	○ B	CHF 63	15%
18		A ●	○ B	CHF 62	10%
19		A ●	○ B	CHF 61	5%
20		A ●	○ B	CHF 60	0%

We applied a similar random payment method in the time task as in the risk task: One of each subject's choices was paid out for real at the corresponding payment date. Average payoffs for the time task amounted to CHF 64.34. Therefore, total average payments for both risk and time tasks summed to more than four times students' opportunity costs, measured by the student assistants' hourly wages.

Since our objective was to elicit discount rates over guaranteed payments, we took special care with the payment procedure: First, every single subject got paid for one of her intertemporal choices all of which entailed a payment of the same order of magnitude. Not paying off everyone may render the stochastic nature of the experimental earnings salient and interfere with the objective of eliciting discount rates over guaranteed amounts of money. The second issue concerns the credibility of payment. In order to control for uncertainty arising from subjects' doubts about experimenter reliability, an official voucher of the Swiss Federal Institute of Technology was issued to them. This payment method was explained in detail in the instructions, and a specimen of the voucher was included in the instruction set.

A third possibly confounding factor are transaction costs. Transaction costs should be the same regardless of the payment date in order to avoid inducing present bias resulting from immediately available cash payments. Therefore, every subject had to make a trip to the cash desk to collect her earnings for the discounting task.[7]

A.3 Econometric Specification

The data elicited in the experiment provide two types of main variables: certainty equivalents ce for risky prospects, and imputed discount rates δ_t for temporal prospects. We first discuss our econometric approach to risky choice and, subsequently, describe the method employed to test for a link between risk preferences and time discounting.

A.3.1 Behavior under Risk

Modeling decisions under risk encompasses two components, a model of behavior on the one hand, and assumptions regarding decision errors on the other hand. Risk taking behavior is modeled by rank dependent utility theory (RDU).

According to RDU, an individual values a two-outcome lottery $\mathcal{L} = (x_1, p; x_2)$, where $x_1 > x_2 \geq 0$, by $w(p)u(x_1) + (1 - w(p))u(x_2)$. The function $u(x)$, with $u(0) = 0$ and $u'(x) > 0$, describes how monetary outcomes x are subjectively valued. The function $w(p)$ assigns a subjective weight to every outcome probability p, with $w(0) = 0$, $w(1) = 1$, and $w'(p) > 0$. The decision maker's predicted certainty equivalent $\hat{ce}$ for this lottery can then be written as

$$\hat{ce} = u^{-1}\left[w(p)u(x_1) + (1 - w(p))u(x_2)\right]. \tag{A.1}$$

In order to make RDU operational, we have to assume specific functional forms for the utility function $u(x)$ and the probability weighting function $w(p)$. Given our objective of describing individual behavior, we choose flexible functional forms for u as well as for w. A natural candidate for utility u is a power function. In its extensive form, as discussed by Wakker (2008), u is modeled as[8]

[7]People entitled to payoffs the next day were issued vouchers immediately after the experiment. All the others received official certificates of indebtedness after the experiment, the vouchers themselves were sent to them by registered mail several days before they could cash them in, so they did not have to worry about forgetting encashment or misplacing their vouchers.

[8]Note that $\ln x$ is not defined for $x = 0$. Therefore, estimation is carried out after shifting all outcomes

$$u(x) = \begin{cases} x^\eta & \text{if } \eta > 0, \\ \ln x & \text{if } \eta = 0, \\ -x^\eta & \text{if } \eta < 0. \end{cases}$$

A variety of parameterizations of probability weighting functions $w(p)$ have been proposed in the literature (Karmarkar, 1979; Lattimore et al., 1992; Tversky and Kahneman, 1992; Prelec, 1998). Since our primary interest lies in common ratio violations we focus on a specific characteristic of the weighting function, subproportionality. Subproportionality means that for a fixed ratio of probabilities the ratio of the corresponding probability weights is closer to unity when the probabilities are low than when they are high (Kahneman and Tversky, 1979). Intuitively speaking, scaling down the original probabilities makes them less distinguishable from each other, thus favoring preference reversals. Therefore, subproportionality maps common ratio violations. Expressed formally (Prelec, 1998), *subproportionality* holds if $1 \geq p > q > 0$ and $0 < \lambda < 1$ implies the inequality

$$\frac{w(p)}{w(q)} > \frac{w(\lambda p)}{w(\lambda q)}. \tag{A.2}$$

As we rely on subproportionality as the crucial characteristic of the probability weighting function, to be estimated for each single individual, we adopt the flexible and empirically well-founded two-parameter specification suggested by Prelec (1998),

$$w(p) = e^{-\beta(-\ln p)^\alpha}. \tag{A.3}$$

For $\alpha < 1$, the function is subproportional everywhere, with the parameter α measuring the degree of subproportionality.[9] A smaller value of α reflects a more subproportional curve. Therefore, this specification enables us to rank individuals according to their their proneness to common ratio violations. The second parameter, $\beta > 0$, is a net index of convexity in that increasing β increases the convexity of the function without affecting subproportionality (Prelec (1998), p.505). Linear weighting is characterized by $\alpha = \beta = 1$.

With regard to error specification we have to reconsider our measurement procedure. In the course of the experiment, an individual's risk taking behavior was captured by her certainty equivalents ce_l for a set of 20 different lotteries $\mathcal{L}_l = (x_{1l}, p_l; x_{2l}),\ l \in \{1, \ldots, 20\}$.

by one unit of money (cmp. Wakker (2008), p.1335).

[9] Prelec (2000) uses the term “Allais paradox index” (p.78).

Since RDU explains *deterministic* choice, actual certainty equivalents ce_l are likely to deviate from the predicted certainty equivalents $\hat{ce}_l$ by a stochastic error ϵ_l, which has to be taken account of. Therefore, we assume that the observed certainty equivalents ce_l can be expressed as $ce_l = \hat{ce}_l + \epsilon_l$, with ϵ_l being normally distributed with zero mean.[10]

Concerning the error variance, we need to account for heteroskedasticity: For each lottery subjects had to consider 20 guaranteed outcomes, equally spaced throughout the lottery's outcome range $x_{1l} - x_{2l}$. Since the observed certainty equivalent ce_l is calculated as the arithmetic mean of the smallest guaranteed amount preferred to the lottery and the subsequent guaranteed amount, the error is proportional to the outcome range. Therefore, the standard deviation ν_l of the error term distribution has to be normalized by the outcome range, yielding $\nu_l = \nu(x_{1l} - x_{2l})$, where ν denotes an additional parameter to be estimated. In total, therefore, four parameters per subject were estimated by maximum likelihood: the curvature of the utility function η, subproportionality and convexity of the probability weighting function α and β, as well as the normalized standard deviation of the decision error parameter ν.

A.3.2 Behavior over Time

Subjects' responses to the intertemporal choice tasks in the experiment provided us with measurements of discount rates δ_2 and δ_4, imputed from the intertemporal tradeoffs between present payments pe and payments x delayed by two and four months, respectively. However, the true underlying discount factors $D(t)$ are defined in terms of utilities, not payoffs.[11] For a temporal prospect $\mathcal{T} = (x, t)$, true discount rates are inferred from the indifference relation $u(pe) = D(t)u(x)$. Measured discount rates, therefore, deviate from the underlying true rates unless u is linear. While in our specification utility curvature affects the level of discount rates but cannot, by itself, induce their decline, it may have a confounding effect on the magnitude of the change in the measured discount rates $\Delta\delta = \delta_2 - \delta_4$: In the presence of nonlinear probability weighting, $\Delta\delta$ gets amplified by the concavity of the power utility function (see Appendix A.7.4). Specifically, the more concave u, the larger the measured difference in the discount rates $\Delta\delta$ if $w(p)$ is not linear. Therefore, we have to control for the degree of concavity η in the regression model.

[10] Since ce is calculated as the arithmetic mean of two neighboring amounts in the choice menu it possibly contains some measurement error. As ce is the dependent variable in the model a measurement error does not pose a problem other than potentially increasing noise.

[11] We assume that the utility of money is a general cardinal function which applies to risky as well as to delayed payoffs. See Wakker (1994) for a justification of this assumption.

A.3.3 Regression Model

We investigate the hypothesized relationship between probability weighting and decreasing discount rates by regressing the difference between the imputed discount rates δ_2 and δ_4, $\Delta\delta$, on a vector of regressors c. In the base model, Model 1, the vector c consists of a constant and the individuals' estimated risk preference parameters: η captures concavity of the utility function, α captures subproportionality of the probability weighting function, and β its convexity. Additionally, we estimate an extended version of the base model, Model 2, by controlling for a set of individual characteristics. In particular, these controls comprise gender (labeled *Female*), age *(Age)*, the logarithm of disposable income per month *(Log-Income)*, a binary variable indicating whether the subject is familiar with investment decisions *(Experience)* as well as the test score for the *Cognitive Reflection Test (CRT)* (Frederick, 2005).[12] This three-question test measures specific aspects of cognitive ability which were found to be strongly correlated with risk taking and discounting behavior.

Unlike the exemplary choice pattern displayed in Figure A.2, a decision maker may have opted for the same option in all rows of the choice menu, which results in a censored observation. In particular, she may have always preferred the smaller sooner option, indicating that her discount rate may lie beyond the maximum value of 95%.[13] As a consequence, the difference between the observed discount rates δ_2 and δ_4 is affected by censoring as well. As ordinary least square (OLS) yields biased estimates in this case, we account for this issue by a censored regression model, described in detail in Appendix A.8. The model has the following form:

$$\Delta\delta_i^* = c_i \underbrace{(\gamma_2 - \gamma_4)}_{\Delta\gamma} + \underbrace{e_{2,i} - e_{4,i}}_{\Delta e_i}, \tag{A.4}$$

where $\Delta\delta_i^*$ specifies the true, but potentially unobserved, difference between δ_2 and δ_4 for individual i, $i \in \{1, \ldots, 112\}$. The error term Δe_i is normally distributed with mean zero and variance σ^2. The interpretation of the regression coefficients $\Delta\gamma$ is equivalent to those of OLS regression, also displayed in the regression output (Table A.3 below).

[12]Summary statistics of the controls are included in Appendix A.11, Table A.4.

[13]A decision maker may also always prefer the later larger option. In this case, we assume a discount rate of 0%. The number of observations at the boundary of the choice menu are listed in Table A.5 of Appendix A.11.

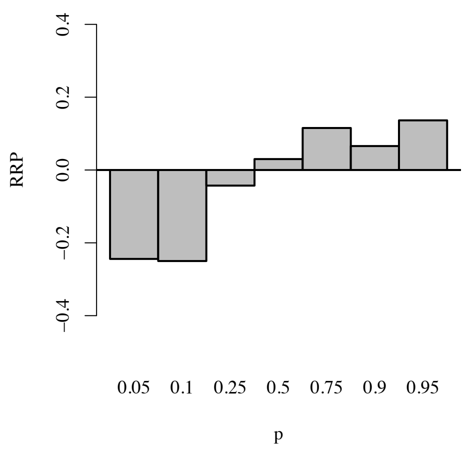
A.4 Results

In the following section we analyze the raw data on risk taking behavior and time discounting, and present the estimates for subjects' probability weights. Finally, we examine the relationship between subjects' sensitivities with respect to changes in probability and delay.

A.4.1 Descriptive Analysis

For the domain of risk taking, Figure B.11 summarizes observed behavior by the median relative risk premia $RRP = (ev - ce)/|ev|$, where ev denotes the expected value of a lottery's payoff and ce stands for the observed certainty equivalent. $RRP > 0$ indicates risk aversion, $RRP < 0$ risk seeking, and $RRP = 0$ risk neutrality. The median relative risk premia, sorted by the probability p of the higher gain, show a systematic relationship between aggregated risk attitudes and lottery probabilities: Subjects' choices display the familiar pattern, i.e. they are risk averse for high-probability gains, but risk seeking for low-probability gains, supporting the existence of nonlinear probability weighting.

Figure A.3: Median Relative Risk Premia *(RRP)*



As far as intertemporal choices are concerned, people's average behavior exhibits decreasing discount rates, i.e. subjects discount less remote outcomes more strongly than more remote ones: The first column in Table A.2 reveals that the discount rates imputed from subject's choices decline on average by 7 percentage points *per annum* when the time horizon is extended from two months to four months. The average data veil heterogeneity as well as the extent of decreasing discount rates, however. Whereas the majority of approximately 54% of all subjects exhibit decreasing discount rates over time, $\Delta\delta > 0$ (second column), about 29% exhibit constant discount rates (third column), and the residual group reveals increasing discount rates (fourth column). Average discount rates of subjects with decreasing discount rates amount to $\delta_2 = 47\%$ *p.a.* and $\delta_4 = 31\%$ *p.a.*, respectively, reflecting a much greater change than do the overall averages.[14]

Table A.2: Average Discount Rates and Risk Parameters

		Subjects with		
	all	$\Delta\delta > 0$	$\Delta\delta = 0$	$\Delta\delta < 0$
	100%	53.9%	29.2%	16.9%
δ_2	0.368	0.465		0.328
	(0.023)	(0.029)	0.213	(0.058)
δ_4	0.299	0.307	(0.045)	0.418
	(0.020)	(0.025)		(0.068)
$\Delta\delta$	0.070	0.157	0	-0.090
	(0.012)	(0.015)		(0.019)
η	0.873	0.808	0.948	0.953
	(0.032)	(0.046)	(0.074)	(0.072)
α	0.505	0.426	0.574	0.634
	(0.021)	(0.027)	(0.040)	(0.063)
β	0.974	0.936	1.064	0.940
	(0.026)	(0.036)	(0.068)	(0.045)
Observations	89	48	26	15

Standard errors in parentheses. Excluding 23 subjects with censored observations.

A.4.2 Risk Preference Parameters

Whereas one of our central variables, change in discount rates $\Delta\delta$, is directly observable, the other one, departure from linear probability weighting, has to be estimated from our data on certainty equivalents.

Individual risk preference parameters η, α and β were estimated on the basis of the econometric model discussed in Section A.3.1. As Table A.2 reveals, the values of the curvature parameter η of the utility function reflect slight concavity or linearity. The average subproportionality index α amounts to 0.505, indicating a pronounced departure from linear probability weighting in line with previous findings (Tversky and Kahneman,

[14]The distributions of the observed discount rates are shown in Appendix A.9.

1992; Gonzalez and Wu, 1999; Abdellaoui, 2000). The average estimates for β lie in the vicinity of one, implying that the respective curves intersect the diagonal at about $p = 1/e$.[15]

The overall picture revealed by our data is consistent with the typical empirical findings: On average, subjects systematically violate linear probability weighting and constant discounting. But the central question, namely whether the degree of subproportionality of probability weighting is associated with hyperbolicity of discounting behavior at the level of the *individual* has yet to be answered.

A.4.3 Relationship between Probability Weights and Hyperbolic Discounting

A first indication of a systematic relationship between probability weighting and discounting can be found in Table A.2. The average estimated subproportionality index α varies substantially across discounting types and exhibits a systematic pattern: α is lowest for the group with decreasing discount rates and highest for the group with increasing discount rates.

This finding is confirmed by the estimates of the regression models. Table A.3 displays the results derived by OLS as well as by the censored regression method. Inspection of the coefficients indicates that censoring seems not to be an important problem: After omitting the 23 censored observations, OLS yields coefficients very close to the estimates of the censored regression model. Furthermore, both specifications (Models 1 and 2) lead to the same conclusion: Subproportionality of probability weighting is significantly associated with decreasing discount rates $\Delta\delta$. Table A.3 shows the estimated coefficient of α to be approximately -0.2. All the respective estimates are significant at the 1%-level and remain robust to inclusion of additional controls. The effect is not only highly significant, it is also quite substantial: A decrease in the subproportionality index α by 0.1 is associated with an increase in $\Delta\delta$ by 2 percentage points *per annum.* In particular, the decline in discount rates is related to the degree of subproportionality, but not to the index of convexity β. We obtained the same order of magnitude for the coefficient of α when we restricted β to be equal to one. Regression coefficients and their standard errors also remain stable when either η or β are deleted from the list of regressors. Moreover, it can be shown that estimates are totally robust to alternative parameterizations of the

[15]Histograms of the parameter distributions are included in Appendix A.10.

Table A.3: Regression Results

Dependent Variable: $\Delta\delta$ ($\Delta\delta^*$)

	OLS[a]		Censored	
	Model 1	Model 2	Model 1	Model 2
Intercept	0.226***	0.279	0.247***	0.321
	(0.063)	(0.228)	(0.057)	(0.225)
η	0.018	0.002	-0.006	-0.022
	(0.042)	(0.043)	(0.039)	(0.041)
α	-0.205***	-0.220***	-0.185***	-0.203***
	(0.066)	(0.074)	(0.062)	(0.075)
β	-0.070	-0.040	-0.074	-0.045
	(0.067)	(0.068)	(0.060)	(0.063)
Female		-0.012		-0.011
		(0.031)		(0.032)
Age		-0.001		-0.002
		(0.007)		(0.007)
Log-Income		-0.013		-0.012
		(0.024)		(0.023)
Experience		0.015		0.020
		(0.032)		(0.033)
CRT		0.021		0.021
		(0.017)		(0.017)
$\hat{\sigma}$	0.123	0.124	0.084	0.082
R^2 or (LogLik)	0.137	0.170	(48.693)	(51.123)
Observations	89	89	112	112
Parameters	4	9	9	19

a) without censored observations.
*** significant at the 1% level.
Bootstrapped standard errors in parentheses (10,000 replications). Bootstrapping accounts for the fact that the regressors α, β and η are estimated quantities.

probability weighting curve as well.[16]

The coefficients of the utility parameter η are not statistically different from zero, either.[17] This result is consistent with our hypothesis that utility curvature *per se* does not impact the extent of decreasing discount rates. Furthermore, none of the other individual characteristics show a significant effect.[18]

An F-test comparing the OLS Model 1 with Model 2 renders a p-value of 0.670, favoring the more parsimonious Model 1, as the controls do not substantially contribute to explaining the variance in $\Delta\delta$.[19] Furthermore, the regression models explain a rather large fraction of total variance: Model 2, for instance, yields an R-squared value of 17%.[20] These findings present conclusive evidence that comparatively more subproportional probability weighting is associated with a stronger decline in discount rates.

A.5 Discussion

The strong and significant correlation between subproportionality of probability weighting and extent of hyperbolic discounting begs the question of whether this relationship can be explained in causal terms. In principle, there are three pathways through which correlation could be generated. First, the tendency towards hyperbolic discounting could cause distortions in probability weights. Since, in experiments, estimates of probability weights are generally based on atemporal choices, i.e. when there is practically no time delay between choice and payment, this possibility can be effectively ruled out. Second, the direction of causality could work the other way round, with proneness to probability distortions inducing hyperbolic discounting. Finally, there could be a third factor driving both types of departures from the standard model predictions. We will discuss the latter possibility first and then turn to the second alternative.

Since the *Common Ratio Effect* and the *Common Difference Effect* pertain to diminishing sensitivity towards probability and delay, respectively, similar cognitive processes may govern the evaluation of risky and delayed outcomes. A natural candidate for a common factor driving both processes is cognitive abilities. Several papers have looked

[16]Results are available upon request.

[17]Nor does an interaction term $\alpha \times \eta$ contribute to explaining variation in $\Delta\delta$.

[18]While not significantly different from zero, coefficients exhibit the expected signs: Females have a slightly more subproportional weighting curve, consistent with previous findings (Fehr-Duda et al., 2006). Both experience with investment decisions and high *CRT* scores are associated with smaller departures from linearity.

[19]The same is the case when the two censored models are compared. A likelihood ratio test of Model 2 against Model 1 renders a p-value of 0.9.

[20]When regressing $\Delta\delta$ exclusively on the socioeconomic variables, R-squared amounts to 3.9%!

into the relationship between cognitive abilities and risk tolerance on the one hand, and between cognitive abilities and patience on the other hand (Frederick, 2005; Benjamin et al., 2006; Dohmen et al., 2007). Generally, they conclude that better cognitive abilities tend to be associated with higher risk tolerance as well as higher patience. For instance, Frederick (2005) finds that students with high *Cognitive Reflection Test* scores gambled significantly more often that did the low *CRT* group, and exhibited lower imputed discount rates, albeit not for choices involving longer time horizons. These previous findings seem to conflict with the insignificant coefficient of *CRT* in Table A.3. Since *CRT* is not correlated with α,[21] the lack of correlation between *CRT* and $\Delta\delta$ indeed suggests that *CRT* scores cannot explain the variance in the hyperbolicity of discount rates. However, we are concerned with sensitivities towards changes in probability and delay and not with measures of average risk aversion and impatience, the focus of previous research. Of course, there could be other factors than cognitive abilities, or aspects of cognitive ability not captured by *CRT* scores, that drive the correlation between subproportionality and hyperbolic discounting. Clearly, this possibility cannot be ruled out and needs further exploration.

Finally, we discuss the last one of our options, direct impact of subproportionality on hyperbolicity of discounting. Many authors have noted before that "[a]nything that is delayed is almost by definition uncertain" (Prelec and Loewenstein (1991), p.784). For instance, a promised reward may, due to unforeseen circumstances, materialize later or turn out to be smaller than expected, or death may keep the decision maker from collecting her reward at all. For these reasons, future consequences are inextricably associated with uncertainty, implying that the decision maker's valuation of temporal prospects not only depends on her *pure* time preference, i.e. her preference for immediate utility over delayed utility, but also on her perception of uncertainty and, consequently, on her risk preferences. In other words, uncertainty drives a wedge between pure time preferences and time discounting.

If this account is an accurate description of intertemporal choice it has far reaching implications for observed discounting behavior, the most obvious one being that behaviorally revealed discount rates will be higher than the rate of *pure* time preference as they include a risk premium. Not surprisingly then, uncertainty has been identified to be an important confound in the measurement of time preferences, which may, at least partly, explain the notoriously high discount rates found in empirical studies (Frederick et al., 2002). The story does not stop here, however. If risk preferences influence time

[21] The Pearson correlation coefficient amounts to -0.0049 (p-value 0.964).

discounting, then people's proneness to probability weighting has to be taken into account as well. Recent contributions have examined the impact of nonlinear probability weighting on discounting behavior theoretically (Halevy, 2008; Walther, 2010). Halevy, for instance, motivated by interaction effects between time and risk found by Keren and Roelofsma (1995) and Weber and Chapman (2005), is concerned with convex probability transformations that can accommodate the certainty effect inherent in the classical Allais paradox. We investigate the more general case of common ratio violations which can be modeled by subproportional probability weights. Subproportionality is not confined to convex functions but may also be present in inverse S-shaped probability transformations, which organize a large part of the empirical evidence. In the following, we show that the degree of subproportionality of probability weights indeed predicts the extent of decreasing discount rates. Furthermore, we derive comparative static results with respect to degree of uncertainty and exemplify the model predictions by graphical illustrations.

A.5.1 A Model of Discounting: The Warped Lens

If the future is perceived as uncertain an allegedly guaranteed delayed outcome $\mathcal{T} = (x, t)$ is effectively evaluated as a risky prospect. Suppose that any future payment is perceived to materialize with a constant per-period probability of contract survival s, $0 < s \leq 1$. Consequently, $\mathcal{T}$ is evaluated as $\mathcal{L} = (x, s^t)$, rendering x with probability s^t and zero otherwise.

As far as the rate of pure time preference is concerned, we adopt the conventional assumption: the rate of pure time preference is characterized by a constant per-period rate $r \geq 0$, resulting in a pure time discount factor ρ equal to e^{-r}.

These assumptions imply that the present equivalent pe of the future payment x, such that the decision maker is indifferent between pe and x, is defined by

$$u(pe) = w(s^t)\rho^t u(x). \tag{A.5}$$

The *effective* discount factor $D(t)$ at delay t equals the weight attached to $u(x)$, i.e.

$$D(t) = w(s^t)\rho^t, \tag{A.6}$$

which depends not only on the *pure* rate of time preference r, but also on the probability of contract survival s as well as on the shape of the probability weighting function w. Clearly, if w is linear, $D(t)$ declines exponentially irrespective of the magnitude of s. If

$0 < s < 1$, the resulting discount factor is lower than for $s = 1$, implying that uncertainty *per se* increases the absolute level of discount rates, but cannot account for discount rates declining over time. In fact, due to uncertainty, discounting would be observed even for a zero rate of pure time preference. If, however, w is nonlinear and $0 < s < 1$, the component $w(s^t)$ distorts the discount factor: As shown formally in Appendix A.7.1, subproportionality of w generates hyperbolicity of $w(s^t)$ in t and, consequently, decreasing discount rates if the future is perceived as uncertain. Metaphorically speaking, the decision maker, when looking into the future, perceives delayed events through the warped lens of probability distortions. A natural extension of this insight is that higher degrees of subproportionality induce more strongly declining discount rates (see Appendix A.7.2). The effective discount factor $D(t)$ also depends on the level of uncertainty s. Higher uncertainty implies more strongly declining discount rates as well. A formal proof appears in Appendix A.7.3.

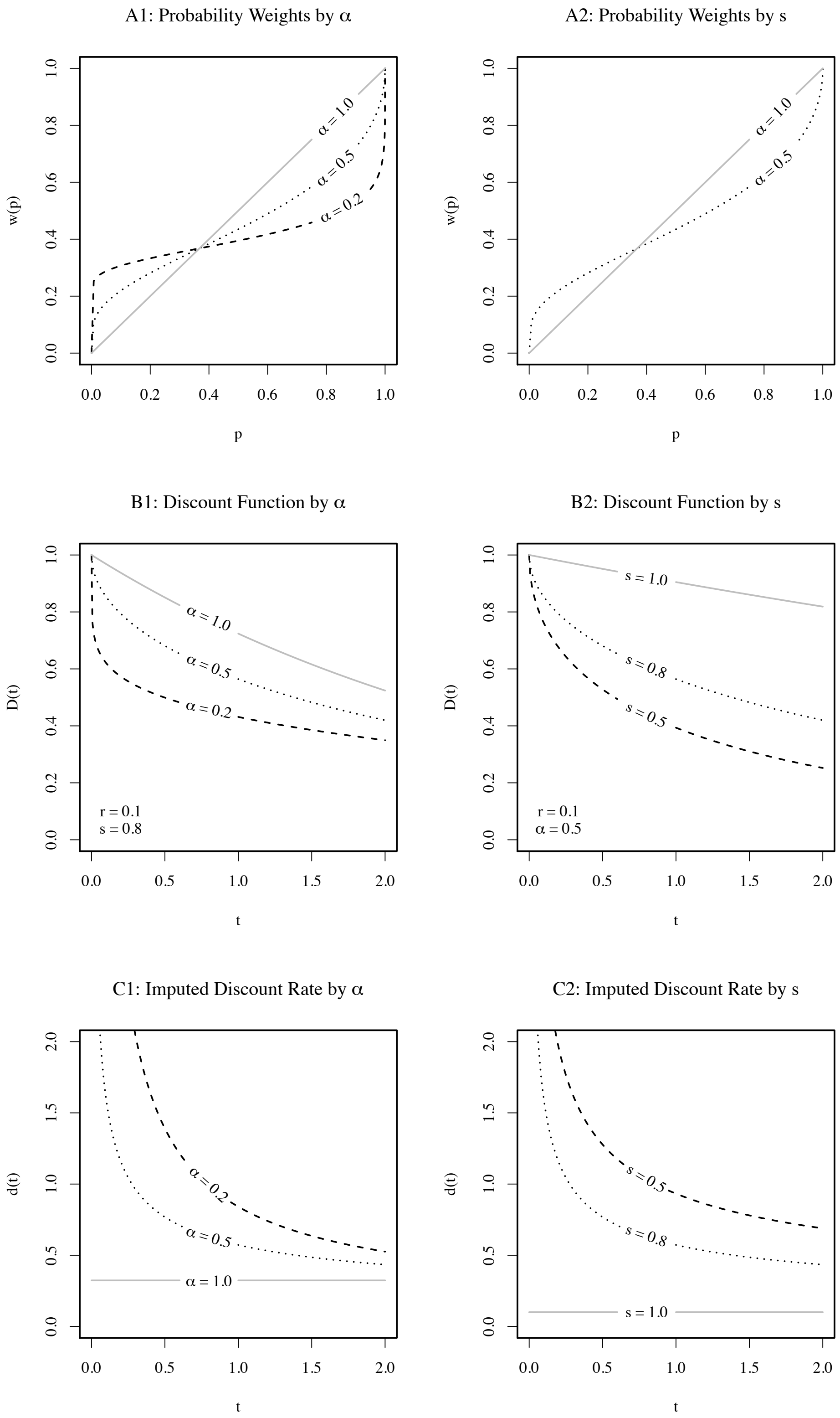
In order to illustrate the predictions of our model, which hold for any subproportional probability weighting function, we demonstrate the comparative static effects of subproportionality α and uncertainty s graphically, using Prelec's specification. The panels on the left-hand side of Figure A.4 show the comparative static effects of varying α, the right-hand side ones are dedicated to varying levels of s.

Panel *A1* of Figure A.4 depicts the probability weighting curves for three distinct parameter values of α, with $\beta = 1$: a medium-sized departure from linearity ($\alpha = 0.5$), as exhibited on average by our experimental subjects, a strong departure from linearity ($\alpha = 0.2$), as well as the limiting case of linear probability weighting ($\alpha = 1$). Panel *B1* of Figure A.4 shows, for each of the three cases of probability weighting, the effective discount factors resulting from Equation A.6 as they evolve over time.[22] For a decision maker with linear probability weighting the discount function, represented by the solid gray curve, is exponential. In contrast, the dotted discount function of a typical decision maker with $\alpha = 0.5$ departs from exponentiality, exhibiting an apparently hyperbolic pattern. By comparison, the decision maker characterized by the most strongly S-shaped probability weighting curve underweights (overweights) large (small) probabilities more strongly than does the decision maker with $\alpha = 0.5$, which leads to an even more pronounced departure from exponential discounting (dashed curve).

Finally, Panel *C1* of Figure A.4 displays the imputed discount rates d_t inferred from $D(t) = e^{-d_t t}$. The solid gray line corresponds to linear probability weighting. Since this

[22]For illustrative purposes, in Figure A.4 r is fixed at 0.1 and s is assumed to be 0.8, which means that 80% of all contracts are anticipated to survive at least one period.

Figure A.4: Probability Distortions and Discounting



decision maker is not prone to probability distortions, her discount rate is independent of time delay and, consequently, constant over time. In contrast to this decision maker, the discount rates of the decision makers with nonlinear probability weights start out at very high levels and then decline sharply. As is evident from comparing the dashed curve with the dotted one, the more subproportional probability weighting function generates a larger decline in discount rates between period 2 and period 1, i.e. the difference $d_2 - d_1$ is greater for higher degrees of subproportionality α. For this prediction to hold the probability of contract survival s needs to be smaller than one. Since people vary in their perceptions of uncertainty our framework predicts a correlation between subproportionality and decreasing discount rates. This is exactly what we find in our data.

Another important insight from our approach concerns the direct impact of uncertainty on discounting behavior. Hyperbolicity of discount rates is crucially influenced by people's perceptions of uncertainty: Increasing uncertainty not only raises the level of discount rates but also exacerbates revealed short-term impatience. Panels *A2* to *C2* in Figure A.4 illustrate this effect for $\alpha = 0.5$, the average index of subproportionality in our data, and $r = 0.1$. When the survival probability s declines from 0.8 to 0.5, the resulting discount function departs more strongly from exponentiality as Panel *B2* shows. Hence, the decrease in discount rates associated with higher uncertainty is more pronounced as well (Panel *C2*).

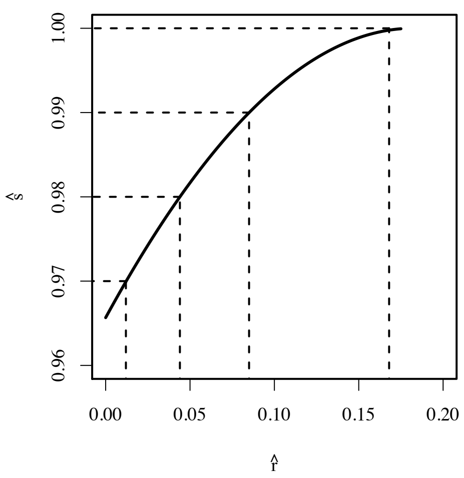
A.5.2 Perceived Uncertainty and the Pure Rate of Time Preference

The model presented in the previous section provides a theoretical underpinning of our empirical finding that the degree of subproportionality α predicts the extent of hyperbolic discounting $\Delta\delta$. Our theoretical framework implies such a relationship *ceteris paribus*, holding constant the other model parameters, specifically the subjective probability of contract survival s and the pure rate of time preference r, both of which are not observable. In our experimental setting with decisions over a short time horizon, the subjective probability of contract survival s should lie very close to unity since mortality risk is very low in our age group of subjects and we took great care to communicate experimenter reliability. One way of checking the plausibility of the theoretical model is to investigate whether, on average, actual choices are indeed consistent with this conjecture, i.e. whether values of s implied by our data lie in the vicinity of one *for a wide range of* *plausible values of the pure rate of time preference r.*

For this purpose, we examine the combinations of s and r that are consistent with the observed average intertemporal tradeoffs between more immediate and more remote payments pe and x. We solve for all feasible combinations of $\hat{s}$ and $\hat{r}$ that are compatible with the observed choices by inserting the estimates for subjects' average behavioral parameters η, α and β into Equation A.5. As is clear from Equation A.5, a higher probability of contract survival needs to be compensated by a higher pure rate of time preference, *ceteris paribus*, to keep individuals indifferent between more immediate and more remote rewards.

As Figure A.5 shows, the feasible $(\hat{s}, \hat{r})$-combinations indeed exhibit a rising profile, with $\hat{s}$ starting out at below 97% *p.a.* and converging to 100% *p.a.*, when the pure rate of time preference increases from 0% to 15% *p.a.* and beyond. For instance, $s = 99\%$ is compatible with $r \simeq 8.5\%$ *p.a.* What this exercise shows is that the data, interpreted within our framework, are consistent with a very high subjective probability that contracts survive at least one year, in accordance with our conjecture. Furthermore, accounting for inherent uncertainty implies rates of pure time preference in a plausible range lying considerably below the observed average discount rates of more than 30% *p.a.*

Figure A.5: Feasible $(\hat{s}, \hat{r})$-Combinations



This suggests that even allegedly guaranteed future outcomes are viewed as slightly uncertain, in line with direct questionnaire evidence provided in Patak and Reynolds (2007). The authors asked respondents to rate their certainties for the same rewards, delayed by 1, 2, 30, 180, and 365 days, respectively, which they had encountered during the preceding choice experiment. The respondents reported ratings that clearly declined with the length of delay. Moreover, using a similar method, Takahashi et al. (2007) found that such subjective probabilities of obtaining delayed rewards decay in a hyperbolic-like manner, consistent with probability weights $w(s^t)$ declining hyperbolically with delay t.

A.6 Conclusion

For several decades, decision research has been dominated by the quest for better descriptive theories of behavior under risk and over time, triggered by a large body of experimental evidence challenging the classical models of choice, expected utility theory and discounted utility theory. Alternative models, accounting for nonlinear probability weighting and hyperbolic discounting, describe behavior much more accurately than do the classical models, at least at the aggregate level. In this paper we address the question whether the better fit of the generalized models is actually a consequence of the same subjects' anomalous behaviors. We present the first evidence that more pronounced systematic departures from linear probability weighting are indeed associated with more strongly declining discount rates at the level of the individual decision makers. This result is robust to inclusion of additional controls as well as model specification. In fact, the only variable explaining a substantial fraction of heterogeneity in individual discounting patterns turns out to be the degree of subproportionality of probability weights.

Several authors have proposed that the existence of matching violations of the classical axioms is not coincidental, but rather reflects the close relationship between risk and delay (e.g. Prelec and Loewenstein (1991)). Some researchers have even argued that the two attributes are virtually the same, but there is no consensus as to which one is the more fundamental of the two. We favor the view that, if there is a hierarchical relationship between them at all, risk is the more likely candidate. To bolster this view, we provide a theoretical model predicting the observed link between probability distortions and decreasing discount rates. For hyperbolic discounting to emerge two factors need to interact: probability distortions and future uncertainty.

Arguably, the future is uncertain by definition. Uncertainty may stem from different sources, either tied to the individual herself, such as lifetime expectancy, or to environmental factors. Lack of contract enforcement and weak property rights, for instance, may make people skeptical that promises will be actually kept. Therefore, institutionally generated uncertainties may induce extreme short-run impatience even if people's pure rate of time preference is low and constant. This insight is important because it implies that revealed behavior may be predominantly driven by environmental factors rather than by the underlying preferences themselves and, consequently, may be amenable to economic policy.

The channel through which uncertainty generates hyperbolic discounting is nonlinear probability weighting, a robust phenomenon in the empirical literature. If probability weighting plays such an important role in risk taking and discounting behavior, the obvious question concerning the source of these probability distortions arises. Unfortunately, little is known about the forces driving probability distortions. A number of theoretical contributions have invoked emotions to explain probability weighting (Wu, 1999; Caplin and Leahy, 2001). Walther (2003, 2010), for instance, rationalizes nonlinear probability weighting by generalizing expected utility theory: He assumes that, in addition to monetary outcomes, the decision maker cares about emotions triggered by the resolution of uncertainty. His approach predicts that, if the decision maker anticipates experiencing elation or disappointment when the actual outcome lies above or below some normal level, she will distort outcome probabilities according to an S-shaped pattern. The more emotional a person expects to be, the stronger will be her departure from linear probability weighting and, consequently, the more pronounced hyperbolic discounting will be. Of course, sensitivity to anticipated emotions is not easily observable, and we have to leave it to future research to investigate whether anticipated emotions or some other factors are the primary drivers of probability weighting.

A.7 Appendix: Formal Proofs

A.7.1 Hyperbolicity

In the framework proposed here, the discount factor $D(t)$ equals

$$D(t) = w(s^t)\rho^t, \tag{A.7}$$

with ρ defined as e^{-r}. In order to establish that subproportional probability weights are sufficient[23] for discount rates to decrease, we define decreasing impatience at t in the following way (Prelec, 2004): Let (x, t) be a temporal prospect paying off x at t with certainty. A preference relation $\succeq$ exhibits *decreasing impatience* if for any $t > 0$, $0 < x < y$, $(x, v) \sim (y, z)$ implies $(y, z + t) \succeq (x, v + t)$.

According to our framework the temporal prospects $(x, 0) \sim (y, 1)$ are evaluated as $u(x)w(s^0)\rho^0 = u(y)w(s^1)\rho^1$. As subproportionality of w implies that $w(s) < w(s^{t+1})/w(s^t)$, deferring the prospects by t periods renders

$$1 = \frac{u(y)w(s)\rho}{v(x)} < \frac{u(y)w(s^{t+1})\rho^{t+1}}{u(x)w(s^t)\rho^t} \tag{A.8}$$

and, therefore, $(y, t + 1) \succ (x, t)$, meeting the requirement for decreasing impatience. ■

In the intertemporal tradeoff between the present and the subsequent period the discount factor equals $w(s)\rho$. At time t, $u(x)$ is discounted by $w(s^t)\rho^t$. Compounding by the initial one-period discount factor $w(s)\rho$ would render $w(s)w(s^t)\rho^{t+1}$ at $t + 1$, but the discount factor effectively amounts to $w(s^{t+1})\rho^{t+1}$ then. Therefore, $w(s^{t+1})/(w(s)w(s^t))$, the wedge between the relative discount factors $D(0)/D(1)$ and $D(t)/D(t + 1)$, provides a measure for the extent of departure from stationarity at t.

A.7.2 Comparative Hyperbolicity

The previous proof shows that, provided that $s < 1$, subproportionality of w engenders hyperbolic discounting. As will become clear shortly, a decision maker with a comparatively more subproportional probability weighting function will also tend to exhibit more strongly decreasing discount rates:

A preference relation $\succeq_2$ exhibits *more strongly decreasing impatience* than $\succeq_1$ if for any intervals $0 \leq v < z, t, \Delta t$ and outcomes $0 < x < y$, $0 < x' < y'$, $(x, v) \sim_1 (y, z)$,

[23] Note that subproportionality is not necessary.

$(x, v+t) \sim_1 (y, z+t+\Delta t)$, and $(x', v) \sim_2 (y', z)$ imply $(x', v+t) \preceq_2 (y', z+t+\Delta t)$ (Prelec, 2004).

In order to examine the effect of the degree of subproportionality on hyperbolicity suppose that the probability weighting function w_2 is comparatively more subproportional than w_1, as defined in Prelec (1998), and that the following indifference relations hold for two decision makers 1 and 2 at periods $v = 0$ and $z = 1$:

$$u_1(x) = u_1(y)w_1(s)\rho \text{ for } 0 < x < y,$$
$$u_2(x') = u_2(y')w_2(s)\rho \text{ for } 0 < x' < y'.$$

Due to subproportionality, the following relation holds for decision maker 1 in period t:

$$1 = \frac{u_1(y)w_1(s)\rho}{u_1(x)} < \frac{u_1(y)w_1(s^{t+1})\rho^{t+1}}{u_1(x)w_1(s^t)\rho^t}. \tag{A.9}$$

Therefore, the subjective probability of contract survival has to be reduced by compounding s over an additional time period Δt to re-establish indifference:

$$u_1(x)w_1(s^t)\rho^t = u_1(y)w_1(s^{t+1+\Delta t})\rho^{t+1}. \tag{A.10}$$

It follows from the definition of comparative subproportionality that this adjustment of the survival probability by Δt is not sufficient to re-establish indifference with respect to w_2, i.e.

$$u_2(x')w_2(s^t)\rho^t < u_2(y')w_2(s^{t+1+\Delta t})\rho^{t+1}. \tag{A.11}$$

Therefore, $(x', t) \prec (y', t+1+\Delta t)$. ■

A.7.3 Uncertainty and Hyperbolicity

In order to derive the effect of increasing uncertainty on hyperbolicity we examine the sensitivity of the departure from stationarity at t, measured by $w(s^{t+1})/(w(s)w(s^t))$, with respect to changing s:

$$
\begin{aligned}
& \frac{\partial}{\partial s}\left[\frac{w(s^{t+1})}{w(s)w(s^t)}\right] \\
= \; & \frac{1}{[w(s)w(s^t)]^2}\left[(1+t)s^t w(s)w(s^t)w'(s^{t+1}) - ts^{t-1}w(s)w(s^{t+1})w'(s^t) - w(s^t)w(s^{t+1})w'(s)\right] \\
= \; & \frac{1}{s[w(s)w(s^t)]^2}\left[(1+t)s^{t+1} w(s)w(s^t)w'(s^{t+1}) - ts^{t}w(s)w(s^{t+1})w'(s^t) - sw(s^t)w(s^{t+1})w'(s)\right] \\
= \; & \frac{w(s^{t+1})}{sw(s)w(s^t)}\left[\frac{(1+t)s^{t+1}w'(s^{t+1})}{w(s^{t+1})} - \frac{ts^t w'(s^t)}{w(s^t)} - \frac{sw'(s)}{w(s)}\right] \\
= \; & \frac{w(s^{t+1})}{sw(s)w(s^t)}\left[(1+t)\varepsilon(s^{t+1}) - t\varepsilon(s^t) - \varepsilon(s)\right] \\
< \; & 0
\end{aligned}
$$

with $\varepsilon(p) = pw'(p)/w(p)$ defined as the elasticity of the probability weighting function w. According to Segal (1987), p. 148, subproportionality holds iff $\varepsilon(p)$ is increasing. As $s^{t+1} < s^t < s$, $\varepsilon(s^{t+1}) < \varepsilon(s^t) < \varepsilon(s)$ and, hence, the sum of the elasticities in the final line of the derivation is negative. Therefore, increasing uncertainty, i.e. decreasing s, entails a greater departure from stationarity and, consequently, a higher degree of hyperbolicity. ∎

A.7.4 Effect of Concavity

In the course of the experiment we cannot observe discount factors at delay t, $D(t)$, directly, however, but infer $\tilde{D}(t)$ from the intertemporal tradeoffs between payments at different dates, i.e. $pe = \tilde{D}(t)x_t$. According to our assumptions, utility is modeled by a power function $u(x) = x^\eta$, $\eta > 0$, which renders $\tilde{D}(t) = D(t)^{\frac{1}{\eta}}$. It follows that

$$
\frac{\tilde{D}(0)/\tilde{D}(1)}{\tilde{D}(t)/\tilde{D}(t+1)} = \left(\frac{D(0)/D(1)}{D(t)/D(t+1)}\right)^{\frac{1}{\eta}} \tag{A.12}
$$

and therefore the observed decrease in discount rates resulting from nonlinear probability weighting gets amplified by $\eta < 1$ and, hence, concavity has to be controlled for in the regression model.

A.8 Appendix: Censored Regression Model

This appendix discusses the way we model the difference in the censored observed discount rates, $\Delta\delta = \delta_2 - \delta_4$, and link it to individual risk preferences.

To relate time discounting to risk preferences, the model assumes the following linear relationship between the discount rate $\delta^*_{t,i}$ of individual $i \in \{1, \ldots, N\}$ over delay $t \in \{$two months, four months$\}$ and a vector of regressors c_i, containing a constant, the parameters of risk preferences, η_i, α_i and β_i, as well as some socioeconomic characteristics:

$$\delta^*_{t,i} = c_i \gamma_t + e_{t,i}, \tag{A.13}$$

where γ_t denotes a vector of slope parameters and $e_{t,i}$ stands for a normally distributed error term with mean zero and variance $\frac{1}{2}\sigma^2$. Under the assumption of nonnegative discounting, the choice menu, depicted in Figure A.2, allows us to directly elicit individual discount rates that lie between 0% and 92.5%. However, if individual i always opts for being paid out at the earlier point in time *(Option A)*, we do not necessarily observe her true discount rate $\delta^*_{t,i}$ as we only know that it amounts to at least 95%. Thus, the elicited discount rates, $\delta_{2,i}$ and/or $\delta_{4,i}$, are censored from above at $b = 0.95$. In the data we observe

$$\delta_{t,i} = \begin{cases} \delta^*_{t,i} & \text{if } \delta^*_{t,i} < b, \\ b & \text{otherwise.} \end{cases} \tag{A.14}$$

This immediately yields the difference in the discount rates over two and four months,

$$\Delta\delta^*_i = c_i \underbrace{(\gamma_2 - \gamma_4)}_{\Delta\gamma} + \underbrace{e_{2,i} - e_{4,i}}_{\Delta e_i}, \tag{A.15}$$

where Δe_i is normally distributed with mean zero and variance σ^2. Consequently, this difference $\Delta\delta^*_i$ is affected by censoring, too, and only observed if both $\delta_{2,i} < b$ and $\delta_{4,i} < b$.

In order to avoid biased estimators for γ_2, γ_4, and σ, the model needs to take the censored nature of the data into account. Therefore, its log likelihood takes on the following form:

$$
\begin{aligned}
\ln L\left(\gamma_2,\gamma_4,\sigma;c,\delta_2,\delta_4\right) &= \sum_{i:\,\delta_{2,i}=b,\,\delta_{4,i}=b} P\left(\delta_{2,i}=b,\ \delta_{4,i}=b\,|\,c,\delta_2,\delta_4\right) \\
&+ \sum_{i:\,\delta_{2,i}<b,\,\delta_{4,i}=b} P\left(\delta_{2,i}<b,\ \delta_{4,i}=b\,|\,c,\delta_2,\delta_4\right) \\
&+ \sum_{i:\,\delta_{2,i}=b,\,\delta_{4,i}<b} P\left(\delta_{2,i}=b,\ \delta_{4,i}<b\,|\,c,\delta_2,\delta_4\right) \\
&+ \sum_{i:\,\delta_{2,i}<b,\,\delta_{4,i}<b} \frac{1}{\sigma}\phi\left(\frac{\Delta\delta_i - c_i\left(\gamma_2-\gamma_4\right)}{\sigma}\right),
\end{aligned}
\tag{A.16}
$$

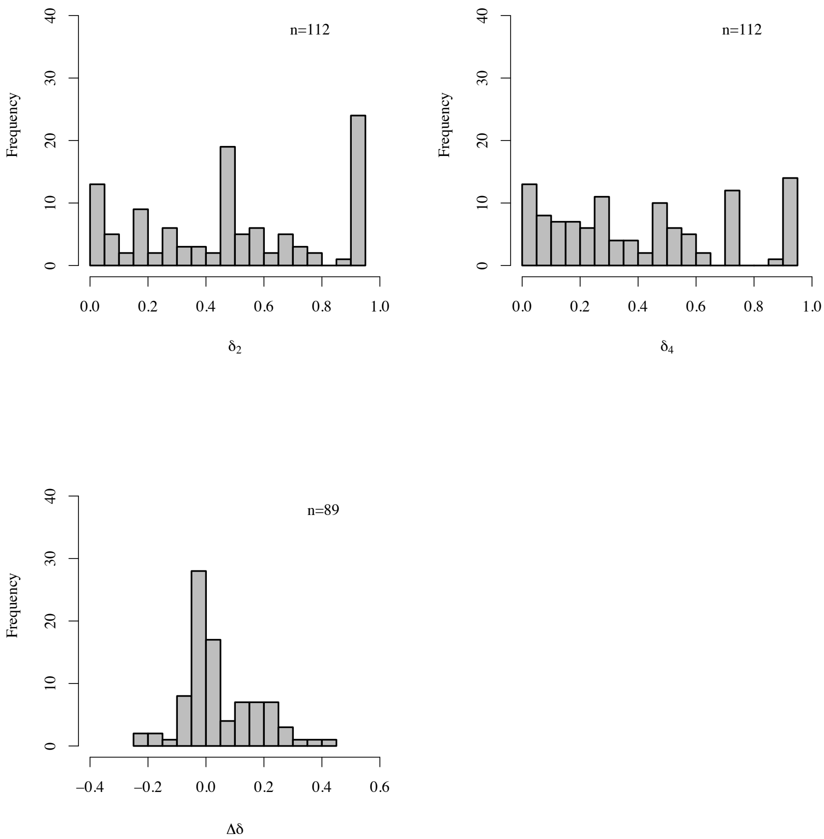
where ϕ represents the standard normal distribution's density and the probabilities P, accounting for the different ways by which an observation may be censored, are given by

$$
\begin{aligned}
P\left(\delta_{2,i}=b,\ \delta_{4,i}=b\,|\,c,\delta_2,\delta_4\right) &= \left[1-\Phi\left(\frac{b-c_i\gamma_2}{\sigma}\right)\right]\left[1-\Phi\left(\frac{b-c_i\gamma_4}{\sigma}\right)\right], \\
P\left(\delta_{2,i}<b,\ \delta_{4,i}=b\,|\,c,\delta_2,\delta_4\right) &= \Phi\left(\frac{b-c_i\gamma_2}{\sigma}\right)\left[1-\Phi\left(\frac{b-c_i\gamma_4}{\sigma}\right)\right], \\
P\left(\delta_{2,i}=b,\ \delta_{4,i}<b\,|\,c,\delta_2,\delta_4\right) &= \left[1-\Phi\left(\frac{b-c_i\gamma_2}{\sigma}\right)\right]\Phi\left(\frac{b-c_i\gamma_4}{\sigma}\right),
\end{aligned}
$$

with Φ denoting the cumulative density function of the standard normal distribution. Numerical maximization of $\ln L\left(\gamma_2,\gamma_4,\sigma;c,\delta_2,\delta_4\right)$ yields the maximum likelihood estimates of $\hat{\gamma}_2$, $\hat{\gamma}_4$, and $\hat{\sigma}$. To obtain the maximum likelihood estimate of $\Delta\hat{\gamma}$ we utilize the invariance property.

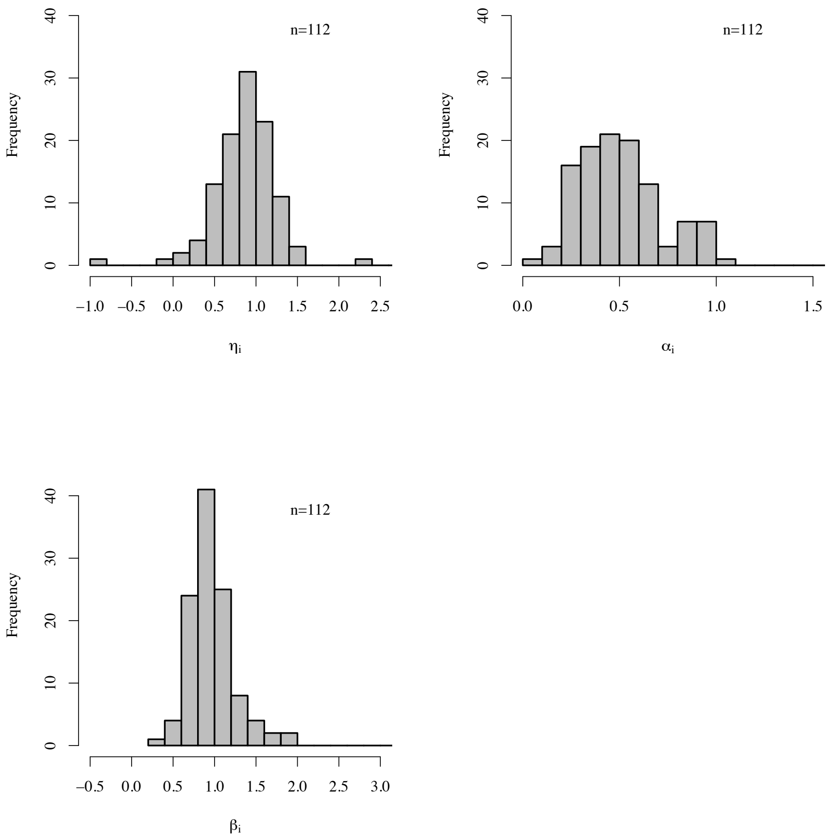
A.9 Appendix: Observed Discount Rates

Figure A.6: Discount Rates δ_2 and δ_4 and Their Change



A.10 Appendix: Estimated Risk Parameters

Figure A.7: Distribution of η, α and β



A.11 Appendix: Controls

Table A.4: Summary Statistics ($n = 112$)

	Type	**Mean**	**Std.Err.**
Female	binary	0.446	0.047
Age	numeric	22.625	0.209
Log-Income	numeric	6.380	0.067
Experience	binary	0.304	0.044
CRT	numeric	2.214	0.082

Table A.5: Number of Observations at the Bounds ($n = 112$)

	δ_2	δ_4
$\geq 95\%$	23	14
0%	2	0

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Appendix B

Rational Planners or Myopic Fools? Liquidity Constraints, Positive Expectations and Anomalies in Intertemporal Choice

This chapter has not yet been published elsewhere.

B.1 Introduction

Many of the most important choices we make involve alternatives with consequences occurring at different points in time. Prominent examples are how much to save for later consumption, when to pay off debts, or in which training to invest. Understanding the drivers of these choices is paramount to predicting individual behavior and market outcomes. In particular, the design of incentive mechanisms, information programs or optimal defaults helping individuals to behave in a more rational way needs to be based on a sound knowledge of where and how to intervene.

This paper contributes to a better understanding of intertemporal choice behavior. It is motivated by a large number of puzzling findings apparently inconsistent with exponential discounted utility, the canonical model of intertemporal choice. Indeed, most solutions proposed to address these issues still suffer from fundamental shortcomings. They typically focus on one single "anomaly" only and fail at predicting the vast number of at least equally important stylized facts. Hyperbolic discounting models (Ainslie, 1975; Herrnstein, 1981; Mazur, 1987; Laibson, 1997; Harris and Laibson, 2008), for example, capture the decline of discount rates in time horizon. They do not, however, provide an explanation for why subject's behavior differs from standard predictions in many more respects.

Empirical evidence documents the following results. First, aggregate behavior departs in systematic ways from exponential discounting (Loewenstein and Thaler, 1989; Loewenstein and Prelec, 1992; Frederick et al., 2002). Quite robust "anomalies" are that discount rates lie far beyond market interest rates, decline in time horizon and in outcome magnitude, and are larger for gains compared to losses. A coherent explanation encompassing all these findings is still lacking.

Second, quantitative result, such as estimated discount rates or the size of these effects, vary tremendously across and within studies (Frederick et al., 2002).[1] This seems puzzling as most experiments are based on similar designs and are conducted among similar cohorts, i.e. subjects that are very homogenous with respect to their education, income and age.

Third, recent longitudinal studies show that behavior is not as dynamically stable as current preference models imply (Sayman et al., 2007; Airoldi et al., 2009). The reason for this finding is still underresearched, but suggests that intertemporal behavior may be influenced by other factors than intrinsic preferences.

[1] In their overview article, Frederick et al. (2002) report discount rates ranging from -6 percent per annum to infinity.

Fourth, many studies find a substantial fraction of subjects exhibiting discount rates increasing in time horizon (Read et al., 2005b; Sayman et al., 2007; Airoldi et al., 2009; Abdellaoui et al., 2010).[2] So far, it is not clear whether this behavior is due to errors, trait or other reasons.

Providing an intuitively appealing and unifying explanation for all these findings is the goal pursued by this article. When individuals want to sustain a smooth consumption path, but are prevented from doing so because they are borrowing constrained and only hold few liquid assets, positive income expectations can have a significant effect on their behavior. Opting for new alternatives materializing at dates when consumption is expected to be relatively low allows them to reach consumption paths which better measure up to their preferences. That such considerations can play an important role in time discounting is supported by empirical evidence. In a study conducted among rural households in developing countries, Holden et al. (1998) find that liquidity-constrained households show much higher discount rates than others.

The idea that liquidity constraints provide a powerful approach for explaining empirical regularities is not new. Deaton (1991) shows that such restrictions can explain many important findings in consumer behavior not captured by most other models. The novelty of our research is to apply this idea to unravel "anomalies" and other puzzling findings in intertemporal choice behavior. We argue that new alternatives are not evaluated in full isolation, as it is usually (implicitly) assumed in empirical studies, but that there are situations where subjective income expectations are reflected in subject's behavior.

We present the following insights. First, we show that all allegedly anomalous patterns naturally arise for a liquidity-constrained, relative impatient consumer with positive, but rational expectations. In fact, all the "anomalies" are closely intertwined with each other. Hyperbolic discounting behavior can even be dynamically consistent. Heterogeneity in the consumers' constraints and subjective expectations provide an explanation for why fairly homogenous subjects substantially differ in behavior.

Second, our approach is first to provide a rationale for so far unexplained behavioral patterns found in empirical data. As time passes, the consumer may face a different life and job situation or may be confronted with an altered economic environment. Her access to liquidity and her expectations about future consumption are therefore likely to change over time. As a result, our model gives a justification for the ostensibly dynamic instability of revealed choice behavior. Furthermore, if the consumer expects her income

[2]Studies reporting similar results are Frederick (1999), Rubinstein (2003), Read et al. (2005a) and Attema et al. (2009).

to substantially decline in the not so distant future, but she is unable to accumulate sufficient liquid assets to smooth away the upcoming low-consumption periods, she exhibits increasing discount rates. Consequently, the distribution of exponential, hyperbolic and counter-hyperbolic types in the population may be largely governed by the liquidity constraints subjects face and the expectations they hold.

Third, in the latter part of the paper we provide empirical support for our approach. We use data from two consumption-savings experiments with monetary incentives, one conducted among junior students (mostly undergraduates) and one among senior students (higher-semester graduates and post-graduates). This data is particularly suitable for testing our conjectures not only because most previous experiments were conducted among students, but also because students are the probably best example for subjects who are limited with respect to their liquidity and who hold significant positive expectations. In accordance with our theoretical model, we find a strong link between income expectations and discounting behavior. A simple binary measure for positive income expectations can explain a large part of the anomalously looking behavior found in our data. Estimation of a structural model further reveals that our approach does well in capturing these systematic patterns. Estimated rates of time preference lie in the vicinity of 10% per annum and are considerably lower than discount rates observed on the descriptive level (larger than 70% per annum). Most interestingly, estimated time preferences are not statistically distinguishable from constant ones. Senior students do not only reveal a more pronounced decline of discount rates in outcome magnitude, but, consistent with their earlier entry into the job market, also seem to expect a larger rise in consumption.

Finally, we discuss the possibility that these behavioral patterns are not necessarily caused by *rational planners*, i.e. liquidity-constrained consumers with positive, but rational expectations, but may also be due to *myopic fools*, overoptimistic consumers who are possibly not liquidity constrained. This alternative explanation for a link between subjective expectations and discounting behavior is motivated by the finding that subjects often hold optimistically biased beliefs when it comes to asses future life events, such as income (Weinstein, 1980, 1987; Dominitz, 1998; Armor and Taylor, 2002). Such consumers will exhibit the typical “anomalies”, but, similar to consumers with hyperbolic preferences, their behavior will not be dynamically consistent. More recent research also indicates that the same mechanism can explain why people suffer from self-control problems. Nordgren et al. (2009) find that people often overestimate their capacity for impulse control, leading them to overexpose themselves to temptations.[3]

[3]There may also be rational reasons for self-control problems. Subjects are required to exert willpower

That different motives can result in observationally equivalent behavior has strong implications for the design of proper policy instruments helping economic agents to behave in a more rational way without harming those that already do. This is of particular importance for paternalistic regulations, programs or regulations helping on an individual basis (Camerer et al., 2003). We reason why mechanisms distinguishing rational planners from myopic fools are ultimately needed to make suitable policy recommendations and propose possible starting points to develop such mechanisms.

The remainder of this article is structured as follows. Section B.2 introduces the theoretical model and its predictions. Section B.3 presents the experiments, the econometric specification and the empirical results. Section B.4 discusses some implications our findings have, provides directions for future research and concludes.

B.2 Model and Predictions

This section introduces the basic model and its predictions. We focus on behavior of a consumer with rational expectations who is limited with respect to her borrowing opportunities and only holds few liquid assets. The particular situation we are interested in is how such a consumer evaluates new alternatives by integrating them in her existing consumption plan. An alternative explanation for a link between subjective income expectations and discounting behavior is given in the concluding section of the paper.

The consumer considered here is characterized by the following preferences.[4] First, she has constant impatience, i.e. at any point in time she has the same strong preference for earlier consumption over later consumption, ceteris paribus. In this case, the consumer attributes the weight $d(t) = \delta^t$, with $\delta \in (0, 1)$, to future consumption utility, where d is her discount function, and $\eta = -\ln(\delta)$ her (constant) rate of time preference. Strotz (1955) shows that such preferences are a necessity for dynamically consistent behavior.[5] We further assume that the consumer is relatively impatient meaning that her rate of time preference η is larger than the real interest rate r. Only if this condition is satisfied, the consumer has a need for additional liquidity and, hence, will demand credit or will dissave from her liquid assets.[6] Second, the consumer has an aversion towards consumption fluc-

in order to resist temptation. If they instead (temporarily) only have limited cognitive resources, they may not be able to exhibit enough self-control.

[4]The assumptions made about preferences are quite standard in the literature (see for instance Fishburn and Rubinstein (1982) or Manzini and Mariotti (2009)).

[5]As we will see in the course of our analysis, however, such preferences are not sufficient to establish dynamic consistency.

[6]See Deaton (1991) for a discussion of this issue.

tuations, i.e. she favors less variable consumption paths over more variable consumption paths, ceteris paribus. Her preferences over outcomes (or consumption quantities) are captured by an instantaneous utility function u satisfying the Inada conditions (Inada, 1963).[7] Comparatively more concave utility functions imply a stronger preference for consumption smoothing.[8]

The consumer has a consumption plan. This plan consists of her expectations about future consumption spendings. We assume rational expectations, i.e., on average, expected consumption coincides with realized consumption.[9] To form a plan, the consumer uses her information about (predictable) future income.[10] Three types of income expectations are distinguished. A consumer who does not expect her income to change within the relevant time horizon is said to hold *constant income expectations*. Similarly, a consumer who expects her income to substantially rise within the relevant time horizon holds *positive income expectations*. The opposite is the case for *negative income expectations*.

Borrowing and saving permit the consumer to transfer income back and forth in time. Likewise, a consumer may dissave from her liquid assets during periods with low income, but save during periods with high income. This allows her to sustain a smooth consumption path even if income is subject to variation. Consequently, if access to liquidity is not restricted, expected short-term income changes should have little effect on actual consumption spendings.[11] It is only expected lifetime income, but not its fluctuations, which governs current consumption (Modigliani and Brumberg (1954), Friedman (1957)).[12]

[7] Viz. the utility function fulfills the following conditions: (1) $u(0) = 0$, (2) continuous differentiability, (3) $u' > 0$ and $u'' < 0$ (concavity), (4) $u'(0) = \infty$ and $u'(\infty) = 0$.

[8] Utility embodies the same preference property as in atemporal settings, where a comparatively more concave utility function expresses a stronger aversion towards variability of outcomes at one single point in time. It is well known that under the assumption of time-additive expected utility (EU) preferences, the coefficient of relative risk aversion and the elasticity of intertemporal substitution are reciprocals. Chew and Epstein (1990) show that this link is not retained for non-EU preferences, however.

[9] More precisely, the actual realization of consumption then deviates from expectations by some symmetric, independent random error, i.e. the expectational error has an expected value of zero, is i.i.d. and uncorrelated with the information the consumer holds at the relevant point in time.

[10] Future income flows are not certain by their very nature. While, for instance, employment is contractable, it is always bound to some residual risk such as a potential job loss or sudden illness. Such considerations play a role when prospective income flows are evaluated. Within her constraints, the consumer thus may form beliefs about the probability distribution of prospective consumption. In this case, the consumption plan may also contain a risk premium, i.e. it consists of certainty equivalents of the underlying distribution. Note that liquidity constraints truncate the income expectation distribution.

[11] For a patient consumer with $\eta \leq r$ similar predictions arise even if there are limited borrowing opportunities (see Deaton (1991) for an elaborate discussion). For $\eta = r$, for example, Schechtman (1976) and Bewley (1977) show that, under i.i.d. or stationary stochastic income processes, consumption converges to the mean of income even if liquidity constraints are present, leading the consumer to reach a perfectly smooth consumption path. Similarly, if $\eta < r$, the consumer will save indefinitely. As Deaton (1991) says it: "[...] saving, not borrowing, is [such a consumers] main concern" (p.1225).

[12] As discussed later, the standard prediction of such a model is that discount rates are constant in

Predictions, however, may differ fundamentally if the consumer is limited with respect to her possibilities to access sources of liquidity. In what follows, we analyze behavior of an impatient consumer who is neither permitted to borrow, nor holds many liquid assets. Nonetheless, as liquidity constraints only hamper borrowing, but not saving, the consumer can still overcome expected future low-income periods. Her limited borrowing opportunities and her limited assets, however, prevent her from smoothing away any enduring low-income period preceding a substantial growth in income. As a result, consumption behavior responds asymmetrically to expected short-term income changes. Consumer's expectations about changes in future income are informative about behavior only if income expectations are positive, but not if they are negative or constant.[13]

That liquidity constraints play an important role in many situations is motivated by the following facts.[14] First, there exists support that a substantial fraction of the population in both, rich and poor countries, is affected by such limitations (Zeldes (1989), Deaton (1991)). Probably most interestingly, the vast majority of empirical evidence on "anomalies" in intertemporal choice is based on experiments conducted among students or residents of developing countries, i.e. subjects who are exceptionally exposed to such constraints. Second, most empirical results rest upon hypothetical choices or hypothetical survey questions. The lack of incentives may lead subjects to ignore borrowing opportunities they actually have. Third, there is direct empirical support for such constraints affecting discounting and consumption behavior. Holden et al. (1998), for instance, find that liquidity-constrained households in developing countries exhibit much higher discount rates than households not facing such constraints. Others (Altonji and Siow (1987), Shea (1995), Drakos (2002)) find that (aggregate) consumption moves asymmetrically with predictable changes in income, a pattern explainable by liquidity constraints, but not myopia.[15] Finally, a certain degree of impatience is required such that a consumer has a need for additional liquidity. If her preferences instead do not comply with these requirements, liquidity constraints will not be binding. There is good reason to believe that

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time horizon and outcome magnitude and do not depend on outcome sign, irrespective of whether the consumer expects her income to be constant, rise or decline in the future. These results are usually tested in empirical settings, where departures from these predictions are interpreted as violations of exponential discounted utility (see Frederick et al. (2002)). This empirical evidence is what originally motivated hyperbolic preference models and alternative discounting models.

[13] Contrarily, if a poor consumer is neither permitted to borrow nor save, responses should be symmetric. The consumption plan then reflects her income expectations.

[14] There are also psychological reasons that may lead a consumer to not borrow or dissave from accumulated wealth, although it is in principal possible. Examples are psychological costs ("sleeping good at night", fearing an even more severe income shock), mental accounting or simply overlooking borrowing opportunities.

[15] Myopia predicts symmetric co-movement of consumption and income.

most subjects are sufficiently impatient to meet this assumption (Frederick et al., 2002).

We now derive our formal model. It is based on standard discounted utility (Samuelson, 1937).[16] We do, however, introduce liquidity constraints. The basic assumptions we make closely follow those in Deaton (1991).[17] Before being confronted with new alternatives, the consumer maximizes her total expected consumption utility such that

$$\max_{(c_0,\ldots,c_\infty)} \mathbb{E}\left\{\sum_{t=0}^{\infty} \delta^t u(c_t) \mid \mathcal{I}_0\right\}, \tag{B.1}$$

where $\mathbb{E}\{\cdot|\mathcal{I}_0\}$ is her expectation conditional on the information available at the decision date 0 and c_t is her *planned consumption* for period t. In period $t+1$, the consumers liquid assets are $w_{t+1} = (1+r)(w_t + y_t - c_t)$, where r is the real interest rate and y_t her exogenous (net) labor income earned in period t. The borrowing restriction takes the form $w_t \geq 0$. The consumer is only allowed to consume out of her "cash-on-hand" $w_t + y_t$, i.e. consumption c_t is bounded from above at $c_t \leq w_t + y_t$ and from below at zero. We further assume that w_t is relatively small. The nonnegative, but small liquid assets the consumer maintains are only intended to insure herself against small and relatively short, negative income shocks. If not stated otherwise, it will therefore be convenient to presume that the consumer can sustain a constant consumption path before and after a substantial rise in income. This is possible because small income fluctuations are easily smoothed away by consuming out of liquid assets.[18]

Our particular interest lies in how new, i.e. previously unanticipated, alternatives are evaluated by the consumer. Thereby, we restrict our attention to singular and certain outcomes.[19] An alternative is a temporal prospect (x, t), where x is the outcome amount and t the outcome date.[20] The consumer integrates new alternatives with her existing consumption plan, her expectations about future baseline consumption c_t. Information

[16]See Koopmans (1960) and Fishburn and Rubinstein (1982) for a more formal treatment of the discounted utility model either for outcome streams or singular outcomes, respectively.

[17]See also Schechtman (1976) and Bewley (1977) who impose similar borrowing constraints.

[18]Small fluctuations of (disposable) income are more likely to occur than substantial changes in labor income. A consumer may therefore anticipate such small oscillations and build up a (more or less small) buffer-stock.

[19]The case where allegedly guaranteed, future outcomes are perceived as inherently uncertain is discussed in Epper et al. (2009). Under this assumption people's proneness to nonlinear probability weighting induces decreasing discount rates and some other interesting links between risk and time behavior.

[20]In the typical intertemporal choice experiment, for example, subjects have to choose between sooner smaller and later larger outcomes. Commonly, it is impossible for the subject to anticipate these outcomes in advance as she does only have limited information about what exactly happens in the lab, what size the payoffs have and when they are carried out.

about liquidity constraints is included in the plan. As limited access to liquidity make it impossible to transfer future income to earlier dates, positive income expectations are reflected in increasing baseline consumption paths. Since saving is still allowed, however, this is not the case for negative income expectations.[21] In other words, liquidity constraints limit the consumer's options to change her ex-ante consumption plan during the evaluation of new alternatives.

The temporal prospect (x, t) is then evaluated relative to the planned consumption, i.e.

$$U_0 = \mathbb{E}\left\{\delta^t \left[u(c_t + x) - u(c_t)\right] \mid \mathcal{I}_0\right\} . \tag{B.2}$$

The availability of new alternatives opens up new possibilities for the consumer to, at least partly, overcome her limited borrowing capabilities and her deficient liquid assets. A liquidity-constrained consumer with the above preferences and positive income expectations can make the huge gap between low and high baseline consumption periods smaller by allocating new outcomes at earlier dates where baseline consumption is expected to be low compared to dates where it is expected to be high. The intuition for this result is straightforward. Due to concavity of the (instantaneous) utility function, marginal utility derived from x additional monetary units of consumption is larger in periods where planned consumption is small compared to when it is large. This result directly follows from Equation B.2. Note that the consumer's relative impatience, $\eta > r$, implies that she does not have an incentive to spread consumption over the remaining low-income periods. Rather, she will consume the entire amount at the point in time it materializes (see Deaton (1991)).

In the following we present the model predictions. As our primary concern lies on explaining observed behavior, we focus on imputed discount rates $\tilde{\eta}$ inferred from predicted present equivalents.[22] We do this for the typical consumer with the above described preferences. Our illustrations and primary results are based on isoelastic utility.[23] In

[21] A consumer with substantial negative income expectations must have enough opportunities to save income during high-income periods. Only this allows her to smooth away the anticipated low-consumption period.

[22] A more precise definition of imputed discount rates can be found in Appendix B.5. Note that all interest and discount rates reported in this paper are per annum rates.

[23] Parameter values are listed in the corresponding figures. It is assumed that $u(z) = z^{1-\rho}$ and $\eta > r$. We set $w_t = 0$ and $c_0 = 0$, such that c_t equals to the expected change in baseline consumption. Note that non-isoelastic utility may induce non-constant discount rates even for neutral expectations or when no liquidity constraints are present. For details, see Appendix B.6.1 and B.6.2.

particular, we are interested in comparing behavior of a liquidity-constrained, impatient consumer holding positive income expectations with a consumer who does not face such constraints, either because she can borrow or is sufficiently patient, or has constant income expectations. According to our model, income expectations only contain information about the consumer's behavior if she is limited with respect to her liquidity.

The next subsections are devoted to illustrate the dependency of imputed discount rates on time horizon, outcome magnitude and outcome sign. Our graphical illustrations assume that the consumer expects an increase in baseline consumption between the point in time she decides ($t = 0$) and the point in time the outcome x materializes ($t > 0$). Additional predictions are sketched out in a special subsection.

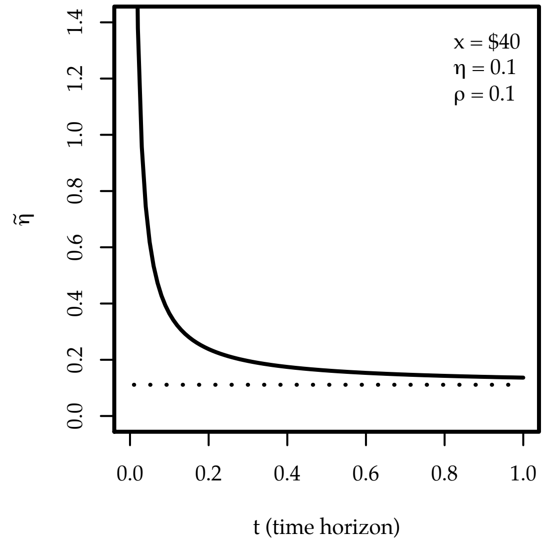
B.2.1 Time Horizon

We first analyze the impact of delay on imputed discount rates. Our model's predictions are a direct result of the basic intuition provided earlier. Liquidity-constrained consumers with positive income expectations allocate new cash inflows at dates where baseline consumption is expected to be relatively low. Since low-income periods precede high-income periods, these consumers show a stronger preference for more immediate payoffs than consumers who do not face liquidity constraints or expect to maintain a constant baseline consumption path, holding all other things fixed. This effect, however, diminishes as the delay grows larger, resulting in imputed discount rates declining hyperbolically in time horizon and converging towards constant long-run rates. Intuitively, high baseline consumption periods gain more weight in the consumers's total welfare, overcompensating the utility generated by consuming in the relatively short-lasting low baseline consumption periods. Hence, for a liquidity-constrained, impatient consumer positive income expectations generate a markup on otherwise constant discount rates. The size of this markup is predominantly driven by how large the consumer expects her income to rise. We prove this formally in Appendix B.6.1.

Figure B.1 illustrates our findings. It plots predicted discount rates for both a consumer with positive expectations (solid curve) and a consumer with constant expectations (dotted line). As can be seen, positive expectations induce decreasing discount rates for the liquidity-constrained consumer. Short-run discount rates appear much larger than long-run discount rates. Although the consumer has constant impatience, her expectations drive a wedge between her time preferences and her discounting behavior.

We obtain very similar results for losses, i.e. consumption reductions rather than

Figure B.1: Time Horizon



$\cdots$ constant expectations (c_t = \$0)
— positive expectations (c_t = \$2)

consumption increases, given total consumption remains nonnegative. However, due to concavity of the utility function, discount rates for losses converge faster towards constant rates.[24] We come back to this result later.

The predictions our model makes accord well with the empirical evidence on intertemporal choice behavior. For monetary rewards, Thaler (1981) observes annualized median discount rates of several dozen to hundred percent. Discount rates decline sharply in time horizon. Similar findings are reported by Benzion et al. (1989) and many others (see e.g. Redelmeier and Heller (1993); Chapman and Elstein (1995); Chapman (1996); Pender (1996); Frederick et al. (2002)). Empirical evidence also supports our long-run predictions. Pender (1996), for example, finds that discount rates far away from the present cannot be distinguished from constant ones. Similar results are also found for negatively signed outcomes (Benzion et al., 1989; Chapman, 1996). Most studies find much lower discount rates in this domain, a finding corresponding well with our predictions.[25]

[24] Subtracting x units from c_t on a concave function leads to a utility loss which is in absolute terms larger than the utility gain when adding x units to c_t. In utility terms, losses then loom larger than gains.

[25] While Thaler (1981) also finds lower discount rates in the loss domain, he does not find supportive evidence for hyperbolic discounting in this domain. His subjects, at least, seem to be less sensitive to changes in time horizon for losses compared to gains. There are a number of possible explanations for this divergent result. First, monetary losses are hard to implement, as it is usually not possible to

One of the most remarkable results predicted by our model, however, is that hyperbolic discounting behavior can be dynamically consistent. This is the case if the consumer's expectations do not differ systematically when reevaluating previously made choices at some later point in time. We prove stationarity under rational expectations in Appendix B.7.

Our findings indicate that hyperbolic discounting is not necessarily a behavioral anomaly caused by time-inconsistent preferences (Thaler and Shefrin, 1981; Laibson, 1997; O'Donoghue and Rabin, 1999; Harris and Laibson, 2008),[26] but can be the result of the consumer's reaction to limited access to liquidity. Allocating new outcomes at dates where these constraints are binding can help to compensate for these limitations. A direct result from this reaction is that revealed discount rates are subject to distortions. Contrary to hyperbolic discounting models, however, our approach not only predicts that discount rates decline in time horizon, but also provides a tractable explanation for a large number of additional, anomalously-appearing findings. The basic distinction between hyperbolic discounting behavior we predict and genuine hyperbolic preferences is that it is the *calendar date* the outcome materializes and not the *temporal delay* which governs how much weight the consumer puts on future consumption utility. As time passes, the temporal distance to the payment date grows shorter in our model, but as average consumption expectations may remain the same and time preferences are constant, this does not necessarily lead the consumer to reverse her revealed preferences. For hyperbolic preference models this is not the case. The two competing models' predictions may therefore differ in dramatic ways, especially when it comes to repeated choices.

Our approach also provides a possible explanation for why many studies find a sub-

force participants to pay money (back) to the experimenter. Appropriate framings are therefore needed. Thaler (1981) encounters this issue by using (hypothetical) monetary fines. Other use, for instance, (hypothetical) debt repayments (Benzion et al., 1989) or (real) punishments, such as electric shocks (Mischel et al., 1969). Obviously, the baseline level subjects consider is likely to strongly depend on the situation they are confronted with. Second, the evaluation of prospective outcomes is often influenced by psychological costs. Such costs may play an exceptional role when subjects are confronted with fines (as in Thaler (1981)). Similarly, when electric shocks or a worsening of health state come into play anxiety or dread during the time waiting for the event to materialize can even induce counter-hyperbolic discounting patterns and negative discount rates (see Loewenstein (1987) for a discussion of savoring and dread motives in time discounting).

[26] Hyperbolic discounting models typically explicitly model *preferences* as the outcome of an internal conflict of interest between a short-run and a long-run self. The result is dynamically inconsistent behavior. Note, however, that not all researchers introducing or applying hyperbolic preference models provide such an intuition. Many consider these models as pure reduced-forms. Mostly these models are motivated by the, at least on aggregate, superior fit they have compared to exponential discount functions (Rachlin et al., 1991; Myerson and Green, 1995; Kirby, 1997) or the fact that many empirical regularities can be explained within such a framework (Laibson, 1997). See Ainslie (1975), Herrnstein (1981) and Mazur (1987) for some earlier proposals of hyperbolic discount functions.

stantial fraction of subjects exhibiting discount rates increasing in time horizon (see for instance Read et al. (2005b); Sayman et al. (2007); Airoldi et al. (2009); Abdellaoui et al. (2010)). If a consumer expects her income to decline substantially in the not so distant future, but is unable to accumulate sufficient liquid assets to overcome the anticipated low-income periods, she exhibits increasing discount rates. The story is the same as above, but this time the consumer is limited with respect to put aside present income for future consumption. Opting for new alternatives materializing during future low-income periods may help her to partially overcome these limitations. As a result, imputed discount rates lie below her rate of time preference. To the best of our knowledge, our approach is the first at explaining this economic relevant, but puzzling finding. We suspect that relatively poor consumers with a marginal propensity to consume out of their "cash-on-hand" close to one and substantial negative expectations are most likely to exhibit such behavior.

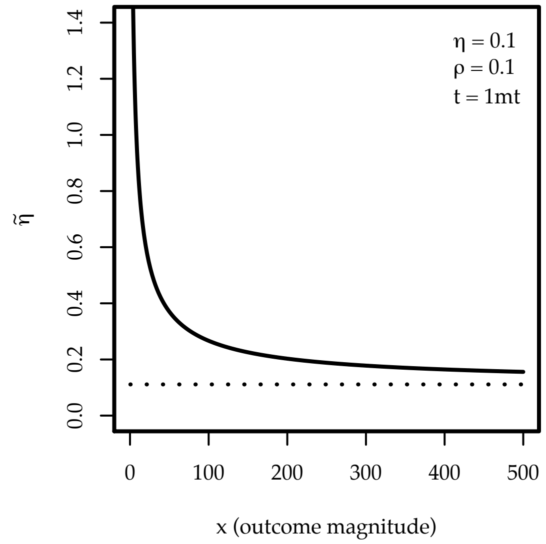
B.2.2 Outcome Magnitude

Our second finding concerns the magnitude effect, i.e. the empirical regularity that smaller outcomes are discounted more heavily than larger ones. This effect directly follows from the consumers's temporal allocation of new cash inflows at dates where marginal utility is largest. As fixed changes in baseline consumption have a higher impact on smaller outcomes compared to larger outcomes, imputed discount rates decline in outcome magnitude. The effect strongly depends on the consumer's marginal rate of intertemporal substitution. It is more pronounced the more future baseline consumption is expected to grow. Negligible or distant positive changes in baseline consumption are unlikely to drive significant magnitude-dependency, however. We derive these results mathematically in Appendix B.6.2.

Figure B.2 depicts our findings. For a liquidity-constrained, impatient consumer with positive expectations, imputed discount rates are substantially larger for small outcomes compared to large outcomes, but they decline as stake size increases (solid curve). This is not the case for a liquidity-constrained consumer with constant expectations (dotted line). Such a consumer is not sensitive to changes in outcome magnitude, but reveals imputed discount rates constant in outcome magnitude. Similar behavior is predicted for consumers who are not liquidity-constrained, irrespective of their expectations.

As long as total consumption does not become negative, we also predict a magnitude effect for losses, i.e. consumption reductions. Again, due to concavity of the utility function the effect is predicted to be less pronounced in this domain. Details follow in the

Figure B.2: Outcome Magnitude



$\cdots$ constant expectations (c_t = \$0)
— positive expectations (c_t = \$2)

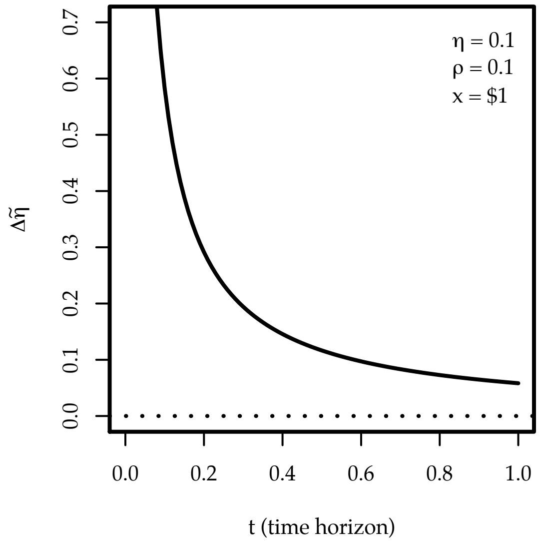
next subsection.

Our predictions dovetail nicely with the empirical evidence provided by numerous studies (Thaler, 1981; Loewenstein, 1987; Benzion et al., 1989; Holcomb and Nelson, 1992; Raineri and Rachlin, 1993; Shelley, 1993; Green et al., 1994a,b; Kirby and Marakovic, 1995; Kirby, 1997; Kirby et al., 1999). For rewards, Thaler (1981) not only finds that median discount rates decrease in outcome magnitude, but also that *"subjects' actions are closer to the normative model, the larger are the stakes"* (p.206). Similar results, although much less pronounced, are also found for losses (Thaler (1981); Benzion et al. (1989); Shelley (1993); Chapman (1996)).

Despite its prevalence and its relevance for understanding choices over small and large stakes, most competing discounting models, such as hyperbolic discounting models, fail to predict the magnitude effect.[27]

[27]For example, the magnitude effect and its interaction with time horizon can help to better understand the purchase of durable goods. When different alternatives are compared, the relatively small, recurring costs often get too little weight compared to the relatively large, up-front purchasing costs. Exception capturing the magnitude effect are the reference-dependent model proposed by Loewenstein and Prelec (1992) and the magnitude-dependent model by Noor (2010). Loewenstein and Prelec (1992) capture many anomalies by imposing additional assumptions on preferences. In particular, they assume that time preferences are hyperbolic and that the value function has a specific form. Our goal, however, is not to provide a descriptive model, but rather a clear and intuitive explanation for departures from

Figure B.3: Outcome Sign



$\cdots$ constant expectations (c_t = \$0)
— positive expectations (c_t = \$2)

B.2.3 Outcome Sign

So far, we found that subjective income expectations can induce asymmetric discounting behavior with respect to differently signed outcomes. The sign effect, i.e. the regularity that gains are discounted more heavily than losses, is predicted for liquidity-constrained, impatient consumers with positive income expectations. The intuition is the following. As the instantaneous utility function is concave, sacrificing consumption always hurts more than increasing consumption by the same absolute quantity yields pleasure. It follows from the previously examined effects that the difference between domain-specific discount rates diminishes as the time horizon or the absolute outcome magnitude becomes larger. As a result, behavior is predicted to be more symmetric the larger the prospect's arguments become, ceteris paribus.

Figure B.3 shows our predictions by plotting the difference of the domain-specific discount rates over time horizon. Liquidity-constrained consumers with positive expectations show asymmetric discounting behavior (solid curve). The asymmetry between gains and losses is largest for short time horizons, but diminishes the more remote the outcome

standard predictions, without violating exponential discounted utility. Noor (2010) ex-ante assumes a magnitude-dependent discount factor and derives the hyperbolic discounting model as a special case.

materializes. Symmetric behavior is predicted for a consumer with constant expectations (dotted line) as well as for a consumer without liquidity constraints. Similar results arise for large-stake outcomes.

Empirical studies such as Yates and Watts (1975), Thaler (1981), Loewenstein (1987), Loewenstein (1988), Benzion et al. (1989), MacKeigan et al. (1993), and Chapman (1996), all find that gains are discounted more heavily than losses. Evidence also seems to indicate that this effect diminishes as the time horizon and the outcome magnitude grows larger (see e.g. the results in Thaler (1981)).

Most competing discounting models do not predict sign-dependency.[28] This can have severe consequences for predicting behavior for choices where both, gains and losses, are involved.

B.2.4 Other Predictions

Beside the prominent anomalies we discussed so far, our model also explains numerous other stylized facts.

First, as all the systematic patterns we examined originate from the consumer's allocation of new cash inflows at dates where baseline consumption is relatively low, they are all inextricably linked with each other. The model predicts that excessive discounting, decreasing discount rates, magnitude- and sign-dependency should not be observed in isolation, but rather co-occur for liquidity-constrained, impatient consumers with positive income expectations. How strong the respective effects are is largely governed by the liquidity constraints the consumer faces, her income expectations, her preference for smooth consumption paths and her impatience. A liquidity-constrained, impatient consumer with an aversion towards consumption fluctuations and comparatively higher income expectations, for instance, should exhibit discount rates decreasing more strongly in time horizon, a more pronounced magnitude effect, and more asymmetric discounting of gains and losses, ceteris paribus. As the time horizon or the outcome magnitude grows larger, however, imputed discount rates approach constant rates, rendering the exponential discounted utility model a good approximation for long-run and large-stake behavior. The empirical evidence we discussed above seems to support our conjecture. These behavioral patterns usually appear conjointly, but they are less pronounced in the long run.

Second, as subjects are very heterogenous with respect to their preferences, their liquidity, their expectations and possibly even their rationality, our model also provides

[28] See Loewenstein and Prelec (1992) for a notable exception. In their model this anomaly arises by ex-ante assuming different elasticities of the value function for gains and losses.

an explanation for the puzzling huge variation within and between studies (Frederick et al., 2002). While most other models assume that behavioral heterogeneity is solely driven by interpersonal differences in preferences, our analyses reveal the importance of other factors for understanding the vast behavioral differences. Subjects may, for instance, reveal different intertemporal behavior because they have different plans or face different environments.

Third, as the consumer's life or job situation may change over time, or the consumer may be confronted with an altered economic environment, liquidity constraints and subjective income expectations are likely to differ at some later point in time. This may explain why recent longitudinal studies find that discounting behavior is so dynamically unstable (Sayman et al., 2007; Airoldi et al., 2009).[29] These studies allow the conclusion that today's preference models, such as hyperbolic discounting, do a rather bad job of predicting behavior. Our approach provides a clear and testable intuition for the apparent dynamic instability of behavior and may help to improve predictions in such settings.

Fourth, as relatively poor consumers are more likely to face liquidity constraints, our model predicts that they should also depart more strongly from standard predictions, holding all other things fixed.[30] Hyperbolic discounting, the magnitude effect and the sign effect should be more pronounced for subjects exposed to such limitations and expecting a substantial rise in income. Students, for example, are likely to exhibit stronger departures from exponential discounting than middle-aged employees. While it is a very robust finding that wealthier subjects reveal more patience (see e.g. Tanaka et al. (2010)), our results make clear that we should be careful in using lab evidence to make behavioral predictions for other demographic groups or even make policy recommendations based upon it.

To sum up, our model predicts and explains a broad number of anomalously looking behaviors in intertemporal choice. What is still needed, however, is an empirical test of our approach. In the next section we provide first supportive evidence for it.

[29] Sayman et al. (2007), for instance, find that subjects shift to the more patient option as they were confronted the second time with the choice they made previously. This pattern contradicts hyperbolic discounting. They do not find such an effect in another study, however. Airoldi et al. (2009) do not find dynamically stable behavior at all.

[30] Poor consumers are likely to consume the most part of their income. Consequently, they may also react to predictable, short-term negative income shocks.

B.3 Empirical Support

In this section we provide empirical support for our approach. As our primary concern is to explain "anomalies" in intertemporal choice, our setup resembles the typical experiment. In particular, our subjects were confronted with choices between sooner smaller and later larger outcomes. Details of the procedure are described in the next few paragraphs. A description of the structural econometric model follows. Empirical results are presented in the last part of this section.

B.3.1 Data and Experiment

Our data stems from two experimental sessions conducted in Zurich (Switzerland) during 2008, the first one conducted with undergraduate students *(juniors)*, the other one conducted with higher-semester graduate and post-graduate students *(seniors)*.[31] For both experiments, participants were recruited from all fields offered at the University of Zurich and the ETH Zurich. Every subject was confronted with the same procedure.[32] In total, we analyzed 110 subjects', 57 juniors' and 53 seniors', responses.

Student data is particularly suitable for testing our model. Most previous evidence reporting "anomalies" in intertemporal choice is based on student data. Students are typically limited with respect to their borrowing opportunities and access to liquidity.[33] Especially students entering the job market soon (in our case the senior group) should hold substantial positive income expectations. For liquidity-constrained subjects, our model makes distinct behavioral predictions for different kinds of expectations.

The experiments consisted of two elicitation tasks, one dedicated to elicit intertemporal choice behavior *(time task)*, the other dedicated to elicit risk taking behavior *(risk task)*. The purpose of the latter task was to identify the curvature of the instantaneous utility function, a goal which cannot be achieved by using intertemporal choice data alone.[34] A detailed description of the method is given later.

[31] Instructions are available upon request.

[32] The only difference between the two sessions was that juniors answered the questions on a computer, but seniors on paper.

[33] The probably second most common subject pool are residents of developing countries. These subjects are very likely to face similar constraints.

[34] Some recent research proposes ways to solve this issue by implementing special, and typically more complicated experimental procedures. Attema et al. (2009), for instance, propose a method to control for the concavity of the utility function without requiring to elicit it. Andreoni and Sprenger (2009), on the other hand, presents an alternative method which allows to measure both, time and outcome preferences. Earlier attempts to accomplish this goal usually first elicited the utility function and then discount rates (e.g. Chapman (1996)). Our design has the advantage of not departing much from classic elicitation tasks.

The tasks appeared in an individualized random order. Subjects were not informed about the content of the second part of the experiment before they had not finished the first one. Both tasks involved the same range of payoffs, lying between CHF 20 and 80.[35] We carefully selected these amounts such that they were large enough to encourage subjects to consider sooner against later alternatives, but small enough so that they were most probably used for daily consumption.[36]

The choice menus subjects were confronted with were very similar for both tasks. We implemented menus containing a list of 20 varying alternatives which had to be judged against a fixed option. To familiarize subjects with the nature of the procedure, the instructions contained examples and trial problems. The experimenters checked the choices in the trial problems to verify that the subjects had comprehended the task. Besides a show up fee of CHF 20, each subject was paid according to one of her intertemporal choices and one of her risky choices selected randomly at the end of the experiment. Subject's compensation for their intertemporal choices was paid out to them at the respective dates when they received the payment in cash by mail. They received their compensation for the risky choices in cash immediately after the experiment. The detailed payment modalities are described below. It took subjects about one hour to complete the experiment, including a questionnaire following the two choice tasks. They were informed about the approximate duration of the experiment.

As our primary interest lies in intertemporal decision making, in the following we focus on describing the time task. To obtain evaluations of delayed rewards, subjects were presented with 20 choice menus, each one involving a specific temporal prospect $\mathcal{T} = (x, t)$. In accordance with the theoretical part of the paper, we held the sooner date fixed for all choices. The choices were framed as standard consumption-savings decisions. A typical choice menu is presented in Figure B.4. The temporal prospect was displayed on the right hand side of the screen *(Option B)*. *Option A* in the choice menu presents a menu of sooner alternatives, ranging from 0 to x. Every subject had to choose her preferred option in each row of the choice menu. In Figure B.4, a hypothetical subject prefers all sooner payments larger than CHF 36 to the later payment, and prefers the later option in the remaining rows. The earlier reward y making the subject indifferent to the delayed reward x is calculated as the arithmetic mean of the two amounts next to her indifference point, amounting to CHF 37.50 in the example here.

[35]CHF 1 $\approx$ USD 0.95 at the time of the experiment. Note that we did not include negative outcomes, as they are much harder to incentivize and would make the experiment considerably more time-consuming.

[36]Table B.5 in Appendix B.8 illustrates that we achieved that goal.

Figure B.4: Choice Menu — Time Task

	Option A **Payment** tomorrow	**Your Choice**	**Option B** **Payment** in 6 months
1	CHF 57	A ⊙ ○ B	Payment of CHF 60 in 6 months.
2	CHF 54	A ⊙ ○ B	
3	CHF 51	A ⊙ ○ B	
4	CHF 48	A ⊙ ○ B	
5	CHF 45	A ⊙ ○ B	
6	CHF 42	A ⊙ ○ B	
7	CHF 39	A ⊙ ○ B	
8	CHF 36	A ○ ⊙ B	
9	CHF 33	A ○ ⊙ B	
10	CHF 30	A ○ ⊙ B	
11	CHF 27	A ○ ⊙ B	
12	CHF 24	A ○ ⊙ B	
13	CHF 21	A ○ ⊙ B	
14	CHF 18	A ○ ⊙ B	
15	CHF 15	A ○ ⊙ B	
16	CHF 12	A ○ ⊙ B	
17	CHF 9	A ○ ⊙ B	
18	CHF 6	A ○ ⊙ B	
19	CHF 3	A ○ ⊙ B	
20	CHF 0	A ○ ⊙ B	

Note, that contrary to previous attempts to elicit time preferences using choice menus (Coller and Williams, 1999; Andersen et al., 2008), we asked people to state an earlier amount making them indifferent to a later fixed amount instead of the reverse. The main advantage of this design is the absence of censoring issues stemming from arbitrarily defining a range of discount rates per sheet. We therefore give subjects the option to choose from the whole range of outcomes. Moreover, we do not display any interest rates in addition to the monetary amounts as the above mentioned studies do. There are a number of reasons for our choice. First, our econometric approach explicitly models indifference between two temporally delayed outcomes. If interest rates and monetary amounts are presented simultaneously, the researcher would need to know on which criteria choices are based in order to specify the error correctly.[37] Second, we do not want to provide an anchor, e.g. a constant interest rate. For the same reason we do not report expected values of risky prospects in the risk elicitation task. We think that the choices displayed here are more natural, and hence more similar to the everyday choices we make.

The set of temporal prospects followed a factorial design combining five equally-spaced delays between two and ten months with four equally-spaced positive amounts between

[37] An error proportional to the outcome amount is *not* proportional to a continuously compounded interest rate.

CHF 20 and 80.[38] Every subject was confronted with the whole set of prospects once. To avoid order effects, the choice menus appeared in an individualized random order.

At the end of the experiment, one row of one choice menu was randomly selected for each subject. The selected amount was paid out as cash by mail at the respective date. On average, subjects earned CHF 51 for the time task, with average payoffs not differing significantly between juniors and seniors.

We took special care with the payment procedure: First, every single subject got paid for her intertemporal choices. Since incentives need to be salient, intertemporal choice experiments tend to be quite expensive. For this reason, experimenters often pay off only a few participants. While selecting one of the subject's choices at random is unavoidable, not paying off everyone brings out the stochastic nature of the experimental earnings and interferes with the objective of eliciting time preferences over guaranteed amounts of money. For this reason, everyone got paid for the time task. A second issue concerns the credibility of payment. In order to control for uncertainty arising from subjects' doubts about the experimenters keeping their promises, a payment certificate signed by the institute's head was issued to them. The actual payment was then sent by mail at the respective date.[39] This payment method was explained in detail in the instructions. Therefore, subjective uncertainty presumably originated for reasons other than potential contract breach by the experimenter.[40] A third possibly confounding factor are transaction costs. Transaction costs should be the same regardless of the payment date in order to avoid inducing present bias in subjects' responses. Therefore, the earliest possible payment date was the next day and not the same day to ensure equal modalities for all possible payment dates.

The second task served to elicit certainty equivalents over a large set of lotteries. Subjects were confronted with 20 risky prospects $\mathcal{R} = (p_1, x_1, x_2)$, summarized in Table B.1. The choices also appeared in individualized random order and were framed as investment decisions.

The choice menus look very similar to those in the time task. Figure B.5 displays an example.[41] Subjects had to choose between a risky payment consisting of a binary

[38] Remember that experimental designs with tradeoffs between two far-future outcomes are not suited to disentangle constant from decreasing discount rates (see Section B.2).

[39] The Swiss mail service has an excellent reputation and is renowned for its reliable and punctual deliveries.

[40] We asked subjects how they assess the risk of not getting the money from the experimenter. Almost everyone (96%) answered that such considerations played no or no relevant role in their decisions.

[41] The elicitation method is extensively discussed in Epper et al. (2009), Fehr-Duda et al. (2010) and Bruhin et al. (2010).

Table B.1: Risky Prospects

$\mathbf{x_1}$	$\mathbf{x_2}$	$\mathbf{p_1}$	$\mathbf{x_1}$	$\mathbf{x_2}$	$\mathbf{p_1}$
80	60	0.10	60	20	0.05
80	60	0.90	60	20	0.25
80	40	0.05	60	20	0.75
80	40	0.25	60	20	0.95
80	40	0.50	60	0	0.10
80	40	0.75	60	0	0.90
80	40	0.95	40	20	0.10
80	20	0.50	40	20	0.50
60	40	0.25	40	20	0.90
60	40	0.75	40	0	0.50

lottery with outcomes $x_1 > x_2$ and probabilities $(p_1, 1 - p_1)$, and a guaranteed payment. Specifically, *Option B* presented a risky payment of CHF 60 with a probability of 75% or CHF 40 with a probability of 25%, and *Option B* offered 20 alternative, but certain amounts. The varying alternatives were sorted in descending order from the highest amount to the lowest amount. The arithmetic mean of the two monetary amounts next to the indifference point on the choice menu provided the certainty equivalent. For the subject in Figure B.5, for instance, the certainty equivalent z amounts to CHF 53.50, implying weak risk aversion.

We also randomly selected one specific choice of each subject at the end of the experiment. However, the relevant amounts were, conditional on the subject's choices, paid out in cash immediately after the experiment. This is important as we intended to elicit the *instantaneous* utility function using the data from this task.

Average payoffs for the risk task amounted to CHF 54. Therefore, total average payments for both time and risk tasks were considerably larger than the usual hourly income of our subjects. They summed to approximately two times their average hourly wage reported in the questionnaire.[42]

The experiments were complemented by a question on subjective income expectations. In particular, we asked subjects to answer the following question: *"Do you expect your income to rise over the next ten months starting from tomorrow? (Example: You expect*

[42]The average per-month income at the subjects disposal amounted to CHF 526 (51) for juniors and CHF 1224 (136) for seniors (standard errors in parentheses). As our subjects were students, however, they only work part-time. Their average workload was 28% (compared to working 8.5h for five days per week (100%)).

Figure B.5: Choice Menu — Risk Task

	Option A **Guaranteed Payment**	**Your Choice**	**Option B** **Risky Payment**
1	CHF 59	A ⊙ ○ B	Payment of CHF 60 in 75% of all cases *and* Payment of CHF 40 in 25% of all cases.
2	CHF 58	A ⊙ ○ B	
3	CHF 57	A ⊙ ○ B	
4	CHF 56	A ⊙ ○ B	
5	CHF 55	A ⊙ ○ B	
6	CHF 54	A ⊙ ○ B	
7	CHF 53	A ○ ⊙ B	
8	CHF 52	A ○ ⊙ B	
9	CHF 51	A ○ ⊙ B	
10	CHF 50	A ○ ⊙ B	
11	CHF 49	A ○ ⊙ B	
12	CHF 48	A ○ ⊙ B	
13	CHF 47	A ○ ⊙ B	
14	CHF 46	A ○ ⊙ B	
15	CHF 45	A ○ ⊙ B	
16	CHF 44	A ○ ⊙ B	
17	CHF 43	A ○ ⊙ B	
18	CHF 42	A ○ ⊙ B	
19	CHF 41	A ○ ⊙ B	
20	CHF 40	A ○ ⊙ B	

a pay raise during this time period.)". Possible answers were *yes*, *no*, and *don't know.* The measure for *positive income expectations* we use in the following is a binary variable constructed from subjects' answers, being one if a subject expects her income to rise during this time period, and zero otherwise. Note that the wording of the question matches the framing of the intertemporal choices. There, we asked subjects to trade off amounts materializing at a fixed earlier date (the next day) against amounts materializing at some later date.

B.3.2 Econometric Specification

One of our primary goals is to track down drivers of systematic departures from exponential discounting. For this purpose, we estimate a structural econometric model, henceforth labeled *expectations model.* It is based on the theoretical model introduced in Section B.2 and allows us to test its core assumptions and to evaluate its explanatory power. Some additional assumptions are needed to make the model operational and identifiable, however. In what follows, we motivate our approach to control for nonlinear consumption utility and introduce the model's functional form and its error specification. To keep things simple, we focus on preferences and expectations aggregated by group of juniors and seniors, respectively. According to our theoretical model, the presence of liquidity constraints should lead positive income expectations to be reflected in subjects' consumption plans. Such expectations should therefore have measurable effects on behavior. Using our structural model we are able to monetize these expectations. Indeed, what we measure corresponds with what subjects were asked for in the question on income expectations.

The econometric specification we employ nests both, the model usually tested in empirical studies and hyperbolic preferences, as special cases. While the former is essentially an exponential discounted utility model with two additional (usually implicit) assumptions, the linearity of utility in outcomes and the evaluation of outcomes in isolation of existing consumption plans, the latter allows us to assess how much of decreasing discount rates is explained by our approach.

The choice of the risk model, on the other hand, is motivated by the empirical literature on decision making in this domain (see e.g. Starmer (2000)). We employ a rank-dependent utility model (Quiggin, 1982) capturing both, preferences nonlinear in outcomes and preferences nonlinear in probabilities. This allows decomposing observed risk aversion into its two components, marginal utility and probabilistic risk aversion. Wakker (1994) motivates this idea and argues that *"utility should describe an intrinsic appreciation of money, prior to probabilities or risk"* (p.3). As we assume that outcomes in the time task are certain and subjects' expectations should already incorporate their risk premia, this view supports the transferability of (riskless) utility between the two domains.[43] In other words, our instantaneous utility function captures subjects' preferences over outcomes, having similar interpretations in both domains. Accounting for such preferences is of particular importance, as the concavity of the utility function can play a decisive role in intertemporal decision making (see Section B.2). By not doing so, one may significantly overestimate the level of impatience, as well as its decrease in time horizon and outcome magnitude.

Our theoretical model postulates that a subject evaluates any temporal prospect $\mathcal{T}_k = (x_k, t_k)$, $k \in \{1, ..., K\}$, relative to her anticipated baseline consumption level (see Equation B.2). For identifiability, we assume that the stationary part of baseline consumption, i.e. baseline consumption shared by all time periods, equals zero. Moreover, as we are only interested in the average rise of baseline consumption between tomorrow and ten months, we only estimate one single parameter c per group. This parameter does not depend on delay. Under these assumptions, indifference between the stated amount $\hat{y}_k$ and a temporal prospect $\mathcal{T}_k$, is established if the following equation holds:

[43] One may relax the assumption that even guaranteed outcomes are perceived as certain (see Epper et al. (2009)). To keep things simple, we ignore this additional dimension here.

$$\hat{y}_k = u^{-1}\left(d(s,t_k)\left[u(c+x_k)-u(c)\right]\right) , \tag{B.3}$$

where d is the discount function with the earlier and later outcome date as arguments. As the earlier outcome date was held fixed at one day over all choices and hence differs from the decision date, we set $s = 360^{-1}$.

In order to identify u, we further need to specify the risky choice model. We use a rank-dependent specification. An individual values any risky prospect $\mathcal{R}_l = (p_{1l}, x_{1l}, x_{2l})$, $l \in \{1, ..., L\}$, where $x_{1l} > x_{2l}$, by

$$V = w(p_{1l})u(x_{1l}) + (1 - w(p_{1l}))u(x_{2l}). \tag{B.4}$$

As above, the function u describes how monetary outcomes are valued. The curvature is now identifiable, however, since the risky prospects contain lotteries with two nonzero outcomes ($x_{1l} > x_{2l} > 0$), and u appears in both model equations (Equation B.2 and B.4). The function w assigns a subjective weight to every outcome probability p. A subject is indifferent between the stated amount $\hat{z}_l$, her certainty equivalent, if

$$\hat{z}_l = u^{-1}\left(w(p_{1l})u(x_{1l}) + (1 - w(p_{1l}))u(x_{2l})\right). \tag{B.5}$$

In order to make the model operational, we have to assume specific functional forms for the discount function d, the instantaneous utility function u, and the probability weighting function w. Since the main objective is to test the model introduced in the main part of the paper, we choose a discount function nesting constant time preferences. However, this function should be flexible enough to capture hyperbolic time preferences as well. To allow for that, we use a function which can reproduce discount rates that are constant, decreasing or increasing in time horizon. Bleichrodt et al. (2009) propose such a discount function suiting our needs.[44] We extend this discount function in a way accommodating the fact that both outcomes lie in the future. We specify

[44] A subpart of this discount function was originally introduced by Prelec (2004). The extension by Bleichrodt et al. (2009) can also account for strong decreasing and increasing impatience. This specification directly nests the exponential reference case. Most other discount functions are limited in these respects. The generalized hyperbola (Loewenstein and Prelec, 1992), for instance, does neither allow for strong decreasing impatience nor for increasing impatience, and does only contain exponential discounting as a limiting case.

$$d(s,t) = \begin{cases} e^{\eta\left(s^{1-\gamma}-t^{1-\gamma}\right)} & \text{if } \gamma < 1 \\ s^{\eta}t^{-\eta} & \text{if } \gamma = 1 \\ e^{\eta\left(t^{1-\gamma}-s^{1-\gamma}\right)} & \text{if } \gamma > 1 \end{cases}, \tag{B.6}$$

where η reflects the level of impatience, and γ how impatience evolves over time. For $\gamma = 0$, η is equal to a continuously compounded discount rate and d takes on the typical exponential form, where $\eta = -\ln(\delta)$ (see Section B.2). For $\gamma > 0$ ($\gamma < 0$) the function exhibits discount rates decreasing (increasing) in time horizon.[45]

We use a similar specification for the instantaneous utility function u. The power-specification has been originally derived by Pratt (1964).[46] It has the following form:[47]

$$u(x) = \text{sgn}(x) \times \begin{cases} |x|^{1-\rho} & \text{if } \rho < 1 \\ \ln|x| & \text{if } \rho = 1 \\ -|x|^{1-\rho} & \text{if } \rho > 1 \end{cases}, \tag{B.7}$$

where concavity is solely captured by ρ, with ρ/x being the Arrow-Pratt index of concavity. For $\rho > 0$ ($\rho < 0$) the function is concave (convex). $\rho = 0$ reflects the special case where utility is linear in outcomes. u links the two Equations B.3 and B.5.

A potential problem when using such an approach to account for nonlinear utility may be that risk aversion is not solely driven by the curvature of the instantaneous utility function. Rather, subjects may systematically under- or overweight probabilities. Neglecting this source of risk aversion would lead to biased estimates of the curvature parameter ρ. Our data is rich enough to separate these two effects. We control for probability weighting using the following two-parameter function (Prelec, 1998):[48]

$$w(p) = e^{-(1-\beta)(-\ln p)^{1-\alpha}}. \tag{B.8}$$

[45] γ/t corresponds to the Pratt-Arrow convexity of the logarithm of the discount function, $\gamma = -t\frac{[\ln(d(t,s))]''}{[\ln(d(t,s))]'}$, a measure for departures from stationarity proposed by Prelec (1989, 2004).

[46] See Wakker (2008) for a more recent discussion.

[47] Note that $\ln|x|$ is not defined for $x = 0$. Therefore, estimation is carried out after shifting all outcomes by $1E-10$.

[48] We reparametrized the function slightly, such that departures from linear weighting are directly testable. A more general form, also allowing for strong subproportionality, can be derived by extending this model similarly as the discount and utility function showed above.

Using this specification, α denotes an index for subproportionality, where the function is inversely S-shaped for $\alpha > 0$. Higher values correspond to stronger departures from linear probability weighting ($\alpha = 0$). On the other hand, β largely governs the elevation of the curve. More risk proneness is associated with larger, positive β's, where the function intersects the identity line at $p = 1/e \approx 0.37$ for $\beta = 0$.

So far, our model only explains deterministic choice. There may be, however, different sources of error, such as carelessness, hurry or inattentiveness, resulting in accidentally wrong answers (Hey and Orme, 1994). As a consequence, the actual indifference amounts are bound to deviate from predicted indifference amounts by an error. That is, an individual i exhibits $y_{ik} = \hat{y}_k + \epsilon_{ik}$ (in an intertemporal choice) and $z_{il} = \hat{z}_l + \upsilon_{il}$ (in a risky choice). We assume that the error terms are independent within each individuals' choices and normally distributed.

We allow for three different sources of heteroskedasticity in the error variance. First, for each prospect, subjects had to consider 20 certain outcomes, which are equally spaced throughout the prospect's range x_k (for intertemporal prospects) and $x_{1l} - x_{2l}$ (for risky prospects), respectively. Since the observed equivalents are calculated as the arithmetic mean of the smallest earlier or certain amount preferred to the temporal or risky prospect and the subsequent amount on the list, the error is proportional to the prospect range. Second, as the subjects may be heterogenous with respect to their previous knowledge, their attention span as well as their ability of finding the correct equivalent, we expect the error variance to differ by individual.[49] Third, temporal prospects may be evaluated differently from risky prospects. Therefore, we allow for task-specific variances in the error term. This yields the form $\tau_{ik} = \xi_i x_k$ for the time task and $\sigma_{il} = \nu_i(x_{1l} - x_{2l})$ for the risk task for the standard deviation of the error term distribution, where ξ_i and ν_i denote the task-specific parameters of individual i. Note that the model allows to test for both individual-specific and task-specific heteroskedasticity by either imposing the restriction $\xi_i = \xi$ ($\nu_i = \nu$), or by forcing $\xi_i = \nu_i$. Both types of restrictions are rejected by their corresponding likelihood ratio test in both samples with p-values close to zero. It turns out, that errors between the two tasks are not significantly correlated (p-values > 0.14 for both data sets).[50] Therefore, we control for all three types of heteroskedasticity in the estimation procedure.

Having discussed all the necessary ingredients, we now turn to the model specification.

[49] In this simple model, individual-specific heterogeneity in expectations is solely captured by individual error variances.

[50] Based on paired sample tests following Pearson's product-moment correlation.

We are interested in the parameter vector $\theta = (b, \eta, \gamma, \rho, \alpha, \beta)'$, where $\theta^{(T)}$ and $\theta^{(R)}$ extracts the relevant parameters of the time and risk equation from the vector θ, respectively. Given the assumptions on the distribution of the error term, the density for the i-th individual can be expresses as

$$f(y_i, \mathcal{T}; \theta^{(T)}, \xi_i) = \prod_{k=1}^{K} \tau_{ik}^{-1} \phi \left(\frac{y_{ik} - \hat{y}_k}{\tau_{ik}} \right) \tag{B.9}$$

for the time task, and

$$g(z_i, \mathcal{R}; \theta^{(R)}, \nu_i) = \prod_{l=1}^{L} \sigma_{il}^{-1} \phi \left(\frac{z_{il} - \hat{z}_l}{\sigma_{il}} \right) \tag{B.10}$$

for the risk task, where $\phi(\cdot)$ denotes the density of the standard normal distribution. The log-likelihood of the model is then given by

$$\ln \mathcal{L} \left(\theta; y, z, \mathcal{T}, \mathcal{R}\right) = \sum_{i=1}^{N} \left(\ln f(y_i, \mathcal{T}; \theta^{(T)}, \xi_i) + \ln g(z_i, \mathcal{R}; \theta^{(R)}, \nu_i) \right). \tag{B.11}$$

The parameters are estimated by maximizing $\ln \mathcal{L}(\cdot)$ with respect to θ using a quasi-Newton method. We do this for each data set separately, which allows a comparison and interpretation of group differences. Confidence intervals are derived by the 2.5% and 97.5% percentiles of the bootstrap distribution (Efron and Tibshirani, 1993). We account for the structure of the data and thus restrict the resampling procedure to draw with replacement from individuals (clustering).

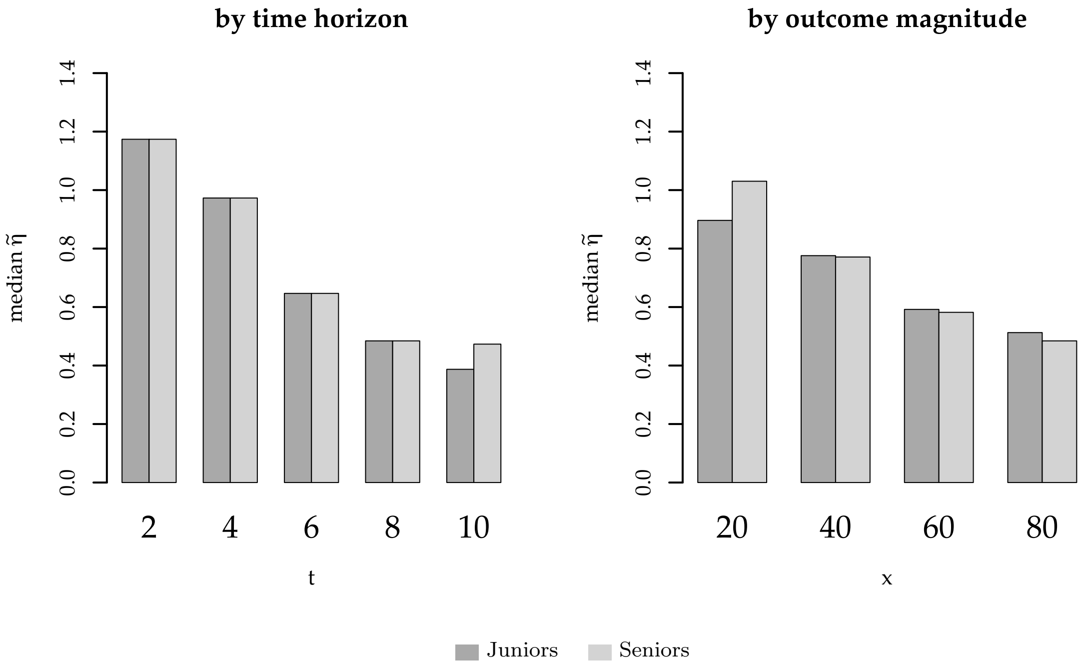
B.3.3 Results

Descriptive Overview

Figure B.6 presents median discount rates for each time horizon and outcome magnitude subjects were confronted with.[51] Both student groups reveal very impatient behavior, with aggregate median discount rates lying at 70.80% per year. That is, discount rates

[51] The discount rates are calculated using the same methodology used in Section B.2 and described in Appendix B.5. We report per annum rates only and account for the fact that subjects trade off between two future outcomes.

Figure B.6: Observed Discount Rates



are considerably larger than market interest rates. These rates decline in time horizon and outcome magnitude, however, lending support for both, the prevalence of decreasing discount rates and the magnitude effect in our data. In summary, observed behavior is consistent with previous experimental evidence on intertemporal decision making. All the typical "anomalies" are found in our data as well.[52]

At first sight, the two groups do not appear to differ substantially. Juniors and seniors show similar median rates. The senior group's discount rates, however, seem to converge more quickly to constant rates. This group also reveals a comparatively more pronounced magnitude effect. On aggregate, their discount rates decline more steeply in outcome magnitude than those of juniors, with the sharpest difference at the smallest outcome. Earlier in this paper we pointed out that for liquidity-constrained, impatient consumers the magnitude effect should be more pronounced the higher subjective income expectations are. Thereby, smaller outcomes are affected more strongly than larger outcomes, ceteris paribus. As we expect the majority of seniors to enter the job market within the next few months, we also expect them to hold higher income expectations, and hence exhibit a more pronounced magnitude effect. This finding therefore fits well to the prediction of our model.

[52] A short descriptive overview for the risk data can be found in Appendix B.9.

Liquidity Constraints and Income Expectations

Positive income expectations should affect behavior if subjects face liquidity constraints. It seems very likely that the vast majority of our subjects are bound by such constraints. Students typically do not have a guaranteed monthly income, neither did they had such an income in the past. As a consequence, they usually do not hold many liquid assets and only have limited borrowing capabilities.[53]

In our experiments we asked subjects whether they had an immediate need for money, i.e. whether they are cash-constrained or not. A considerable fraction of 46.36% of all students reported that they were so.[54] We use this binary variable as a proxy for liquidity constraints.

Figure B.7 illustrates our findings. For each time horizon it plots discount rates for subjects facing different constraints and holding different income expectations. Aside from a few exceptions, *qualitative* behavior between subjects facing and not facing cash constraints does not differ greatly. How discount rates evolve in time horizon seems to depend strongly on the income expectations subjects hold, however. In particular, we find the following. First, as predicted by our model, subjects without positive income expectations reveal discount rates which remain for the most part constant in time horizon (see Panel I and III).[55] The most salient difference between these figures is that subjects facing cash constraints show much higher discount rates. Obviously, having an immediate need for money should lead to more impatient behavior.[56] Second, we also find clear evidence for decreasing discount rates for cash-constrained subjects holding positive income expectations (Panel IV). Both, juniors and seniors, exhibit such patterns. This is one of

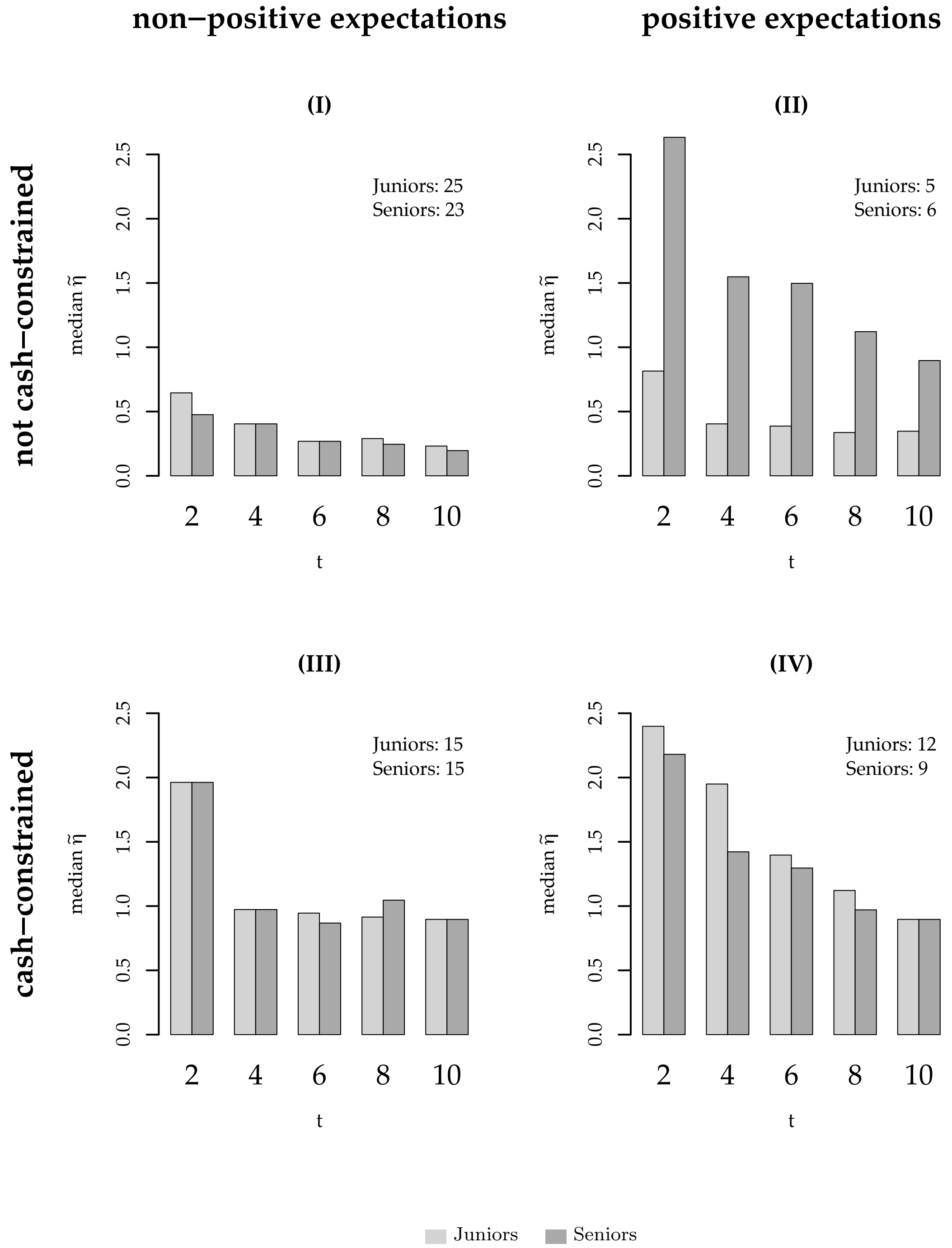
[53]Their limited income does not only prevent students from accumulating much liquid assets. As banks generally do only issue credit cards to account holders if they can document a guaranteed monthly income, they may also have problems accessing liquidity through theses channels. Deaton (1991), for instance, points out that *"[...] there has always been somewhat of a puzzle in the consumption literature as to why individuals who anticipate substantial income growth (e.g. students) and who have a preference for smooth consumption [...] do not borrow large sums in early life. While there are a number of possible answers, particularly at the aggregate level, the existence of borrowing limitations has always been a likely explanation"* (p.1236).

[54]For the two student groups, these fractions are nearly the same (47.37% for juniors and 45.28% for seniors). Standard errors are 4.78 (all subjects), 6.67 (juniors) and 6.90 (seniors).

[55]Both groups exhibit considerably larger rates for the shortest delay in Panel III. While this behavior can be traced back to single individuals, we can only speculate about its causes. One possible explanation may be that our experiments took place some weeks before the spring break. Cash-constrained subjects who have plans for their holidays, but do not expect a rise in their income soon, may favor payments before the holidays, but may still not be willing to sacrifice much of the outcomes in the longer-delay choices.

[56]While these findings support our model it is clear that there may be a reverse causality: More impatient people may be more likely to face cash constraints as they have a higher marginal propensity to consume.

Figure B.7: Cash Constraints and Income Expectations



our model's central predictions. The senior group exhibits a similar behavior in Panel II. Not being cash-constrained does not mean that these subjects have full access to liquidity, however. As we expect senior students to enter the job market soon, and hence to hold much higher income expectations than juniors, a likely explanation is that the expected rise in income is simply too large to be smoothed away. As predicted for subjects without liquidity constraints, juniors show rates which hardly decline in time horizon. Note that the group in Panel II is by far the smallest of the four displayed in Figure B.7. Only five (six) subjects out of 57 (53) are not cash-constrained and simultaneously hold positive income expectations.

To keep things simple and to have a sufficient number of subjects in each group, our subsequent analyses focus on the expectation dimension. This seems adequate as the most striking differences are driven by income expectations rather than cash constraints.[57] Moreover, we are confident that the large majority of our students is limited with respect to access sources of liquidity. For these subjects, positive income expectations should drive departures from standard predictions. Equipped with our binary measure for income expectations, we are able to test this hypothesis explicitly.

Table B.2: Positive Expectations

Group	**Fraction**	**Std.Err.**
Juniors	0.298	0.061
Seniors	0.283	0.062

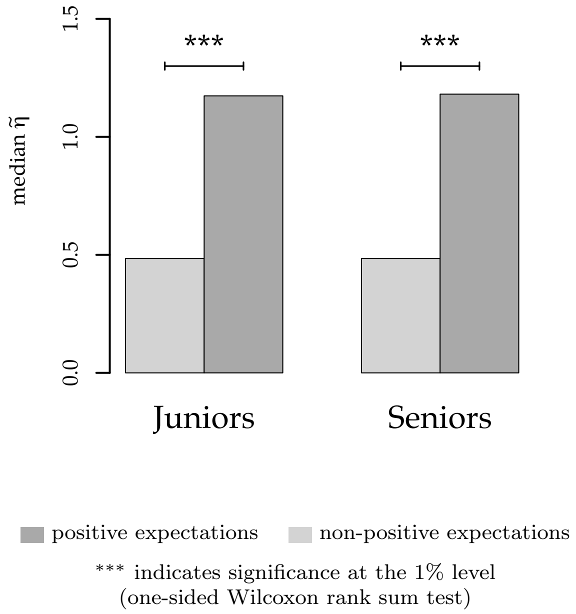
About 30% of our subjects reported that they expected their income to rise in the time period between the next day and ten months into the future (see Table B.2). Assuming standard preferences, i.e. diminishing marginal utility of consumption and impatience, our model predicts that these subjects should reveal higher discount rates which decline more strongly in time horizon and a more pronounced magnitude effect if they are liquidity-constrained compared to subjects not facing such constraints or not having such expectations.

We first illustrate our main results on income expectations graphically. Later, we show linear regressions confirming our findings.

Figure B.8 shows our first result. On aggregate, students with positive income expectations (dark bars) exhibit much higher impatience than students without such expectations (light bars). Both, juniors and seniors expecting an income increase within the relevant time horizon, show much larger discount rates (117.38%) compared to those who do not (48.44%). A nonparametric test indicates that the difference is indeed statistically highly significant for both groups (one-sided Wilcoxon rank sum test, p-values close to zero).

[57] The alternative interpretation we provide in the concluding section gives a possible explanation for this finding.

Figure B.8: Positive Expectations



What is really surprising, however, is how large actual differences between discount rates of the two expectation types are, when bearing in mind that our indicator for subjective income expectations is quite a crude one.

Our second result concerns decreasing discount rates. The upper panel of Figure B.9 depicts clear support for our model. It reproduces what we saw in Figure B.7, but aggregates over the two types with different constraints. Observed discount rates decline much more steeply in time horizon for subjects holding positive income expectations compared to those who do not. This result supports the conjecture that subjective income expectations drive hyperbolic discounting behavior in our data.

While arguably much less pronounced, subjects without positive income expectations still show discount rates declining in time horizon.[58] There are a number of possible explanations for this result. First, we only know whether subjects have positive income expectations or not. Obviously, these expectations are unlikely to be perfectly correlated with subjects' consumption plans. For example, small increases in future income expected soon are still easily smoothed away by dissaving from liquid assets.[59] As we will see later,

[58]This is also visible in Figure B.7, Panel I.

[59]In a similar fashion, subjects may expect their future expenditures or savings ratio to change.

there is indeed evidence for a gap between measured and estimated expectations. Second, these discount rates are still confounded by nonlinear outcome utility. The effect may be significantly biased upwards if instantaneous utility is actually concave. Results reported later indicate that this is also the case here.

Our theoretical model also posits a link between income expectations and the magnitude effect. How strong prospective outcomes are discounted is largely governed by *how much* and *when* the agent expects her income to rise. Individual differences may therefore play an important role in whether such an effect can be found on the aggregate level.

The lower panel of Figure B.9 juxtaposes median discount rates for the two expectation types conditional on outcome magnitude. While we found clear evidence for the magnitude effect on the aggregate level (see Figure B.6), it seems that our income expectation measure is not rich enough to draw clear conclusions here. As subjects are very heterogenous with respect to their income expectations, this finding is likely a result of aggregating over subjects.[60] Analyzing data of two student groups differing with respect to their expectations, however, allows us to test this hypothesis without requiring finer measures on income expectations. In the descriptive part, we found that senior students exhibit a more pronounced magnitude effect. This group should therefore also hold higher consumption expectations. The econometric model we estimate quantifies these expectations for both groups, and hence provides an answer to this question.

The regression results listed in Table B.3 further support the results we obtained graphically. In the Columns I, we regress observed discount rates on positive income expectations, arguments of the temporal prospects and their interaction. This allows us to test how discount rates depend on these variables.[61]

First of all, the relationship between revealed impatience and income expectations is both, quantitatively and statistically significant. Discount rates for subjects with positive income expectations are on average about 88.7 (67.2) percentage points larger than subjects without positive income expectations (about 82.6% (80.1%) per annum). Income expectations also have considerable explanatory power. As additional regressions in Table B.7 (Appendix B.10) show, about 15-20% (!) of the variation in individual median discount rates can be attributed to differences in this binary measure of income expectations.

Second, the regression results also highlight that the large part of the decline of dis-

[60] Another reason is that we use a very simple measure which only allows to keep subjects expecting a rise in their income apart from those that do not.

[61] As we expect subjects to vary greatly in their impatience, we allow them to differ in this respect (random intercept term).

Table B.3: Regression Results

Dependent Variable: Observed Discount Rate $\tilde{\eta}_{ik}^{(1)}$

	Juniors		**Seniors**	
Coefficient	I	II	I	II
Intercept$^{(2)}$	0.826***	0.242	0.801***	1.071***
	(0.113)	(0.909)	(0.113)	(0.688)
Positive Expectations	0.887**	0.977***	0.672**	0.716**
	(0.318)	(0.344)	(0.224)	(0.239)
Time Horizon (in months)$^{(3)}$	-0.102***	-0.102***	-0.063***	-0.063***
	(0.021)	(0.019)	(0.011)	(0.012)
Time Horizon × Pos. Expect.	-0.097**	-0.097**	-0.095**	-0.095**
	(0.045)	(0.046)	(0.040)	(0.040)
Outcome Magnitude (in CHF 10)$^{(3)}$	-0.051***	-0.051***	-0.082***	-0.082***
	(0.014)	(0.016)	(0.013)	(0.014)
Outcome Magnitude × Pos. Expect.	-0.032	-0.032	0.007	0.007
	(0.034)	(0.035)	(0.035)	(0.040)
Log-Income$^{(4)}$		0.163		0.002
		(0.156)		(0.086)
Investment Experience		-0.152		0.006
		(0.488)		(0.244)
Female		-0.312		-0.002
		(0.253)		(0.221)
CRT Score		-0.443		-0.457
		(0.337)		(0.339)
$\sigma_{\text{Intercept}}$	0.875	0.840	0.643	0.627
$\log \mathcal{L}$	-1268	-1268	-903	-906
BIC	2599	2626	1869	1901
Subjects	56	56	50	50
Observations$^{(5)}$	1101	1101	1000	1000

*** (**) indicates significance on the 1% (5%) level; standard errors (in parentheses) are based on a cluster bootstrap using 999 replications

1 discount rate observed by subject i for temporal prospect k

2 random intercept

3 we subtracted the mean value from both prospect arguments, i.e. 6 months (time horizon), CHF 50 (outcome magnitude)

4 the exact transformation is log(income+1) as there were subjects with zero income; income was measured in CHF; summary statistics for all variables are listed in Appendix B.10, Table B.6; qualitative results remain the same when restricting on subsamples of the data, such as gender

5 subjects with missing observations in income and the CRT score are omitted

– estimation procedure accounts for prospect-specific heteroskedasticity

– we do not include higher-order interactions or interactions with controls as their coefficients were either insignificant or did not change the results

– using binary variables for the levels of time horizon and outcome magnitude instead of the original variables leads to qualitatively similar results count rates in time horizon is explained by positive income expectations. Subjects holding such beliefs are about twice as sensitive to changes in time horizon compared to the reference group. On average, their discount rates decline by 19.9 (juniors) or 15.8 (seniors) percentage points when outcomes are delayed by one more month. In comparison, subjects not holding such expectations still show discount rates depending negatively on time horizon, but these rates decline much less sharply, indicating that at least our simple proxy measure is not able to explain the full extent of decreasing discount rates.

Third, the results concerning the magnitude effect comply with our findings above. Increasing the outcome by CHF 10 leads to an average decrease in discount rates by only 8.3 (juniors) or 7.5 (seniors) percentage points. Due to the same reasons already mentioned, however, our measure for income expectations is not capable of explaining much of the decline of discount rates in the outcome dimension.

The Columns II show similar regressions, but include some additional controls. In particular, we control for income, experience with investment decisions (dummy), gender and cognitive ability (cognitive reflection test (CRT) score (Frederick, 2005)).[62] As it turns out, these controls do not contribute much to explaining the variation in discount rates. None of these variables is statistically significant and some even do not show very coherent results for both groups.[63] Moreover, adding these variables to the regression does not change the coefficients for positive income expectations much, and hence do not affect our results.

Parameter Estimates

The discount rates we analyzed so far do not only contain subject's preference for immediate over delayed consumption, but are potentially also confounded by other factors, such as subjects' aversion towards consumption fluctuations and their subjective income expectations. On the observational level, it is therefore not possible to get a clear understanding of the drivers of intertemporal choice behavior. Moreover, the methodology we used so far does not allow to quantify group-specific expectations. In what follows, we resolve these issues by presenting estimation result of the structural econometric model introduced in Section B.3.2.

Table B.4 presents point estimates (*p.e.*) and the corresponding 95% confidence inter-

[62] We did not include age or semesters as controls because there is almost no variation within the two student groups.

[63] A likelihood ratio test suggests that the restricted model should be preferred. Regressing individual median discount rates only on the controls results in R-squares near 2%. Results are available on request.

Table B.4: Estimation Results - Expectations Model

	Juniors			Seniors		
	$p.e.$	$q_{0.025}$	$q_{0.975}$	$p.e.$	$q_{0.025}$	$q_{0.975}$
c	3.697	0.709	9.473	5.414	2.918	19.515
η	0.118	0.071	0.306	0.086	0.023	0.327
γ	-0.008	-0.650	0.708	-0.325	-1.547	0.302
ρ	0.237	0.177	0.296	0.198	0.140	0.261
α	0.497	0.419	0.576	0.614	0.556	0.670
β	0.006	-0.064	0.058	0.102	0.000	0.192
$\ln \mathcal{L}$	-7163			-6769		
BIC	15168			14318		
Subjects	57			53		
Observations	1117			1060		
Parameters	120			112		

95% confidence intervals are derived by the percentile bootstrap method with 9999 replications. The resampling procedure accounts for the panel structure of the data (clusters). Parameters include additional estimates for $\hat{\xi}_i$ for domain- and individual-specific error variances.

vals ($q_{0.025}$, $q_{0.975}$) for the two groups' model parameters. The upper three rows contain the expectation and time preference parameters, the rows below the utility function and risk preference parameters. For both student groups we find concave instantaneous utility and a positive shift in the baseline level. The relevant parameters ρ and c are significantly larger than zero, indicating that the basic assumptions we made in the theoretical part of the paper comply with our data.

There are no substantial differences between the curvatures of the two groups' utility functions. Both show similar concavity. Our estimation results, however, point out how important it is to isolate marginal utility from risk aversion observed in lottery choices. Not doing so would lead to substantially biased estimates for the rate of time preference and hyperbolic discounting. The estimates for the probability weighting functions reveal that there is considerable probabilistic risk aversion in our data. We find significant probability distortions ($\alpha > 0$) and systematic underweighting of probabilities ($\beta \approx 0$) by both groups, results which are in accordance with previous findings on risk taking behavior (Tversky and Kahneman, 1992). Seniors show a weighting curve with a more pronouncedly inverted S-shape than juniors do, but they seem to underweight probabilities less systematically.[64]

In accordance with our model, we interpret c as the average growth of baseline con-

[64]Figure B.12 in the Appendix plots group-specific probability weighting functions against each other.

sumption expected to occur between the next day and ten months into the future. As we suspected earlier, estimated expectations are much larger for seniors, the group of higher-semester graduate and post-graduate students, compared to juniors, the undergraduate students. This finding dovetails nicely with our descriptive results: On aggregate, seniors show a more pronounced magnitude effect. Our model links the comparatively higher expectations to the comparatively stronger effect for this group. The average expectations of this group are estimated to about CHF 5.41, which is considerably larger than those of the junior group amounting to CHF 3.70. In accordance with the information provided to participants (see Section B.3.1), we interpret these amounts as per hour expectations. Average expectations are then equivalent to a rise in monthly consumption by CHF 919.70 (seniors) or CHF 629.00 (juniors).[65] Hence, seniors show expectations that are on average approximately 1.5 times larger than those of juniors. The confidence intervals, however, overlap, indicating that expectations of the two groups do not differ significantly. Nevertheless, there seems to be much heterogeneity with respect to subjective expectations, something one should take special care of when predicting choices.

A crucial question is how much of the findings on the observational level can be explained by our model. It is relatively easy to answer this question for the size of discount rates and their decline in time horizon. Our descriptive analyses revealed rather large median discount rates for the two student groups. These rates amount to 70.8% per year. Estimation results of the expectations model, listed in Table B.4, also contain estimates for the rate of time preference η. These rates lie in the vicinity of 10% per year for both groups, and hence are much closer to market interest rates. On average, seniors seem to be a little bit more patient than juniors. They exhibit a rate of time preference of 8.6%, compared to the 11.8% of juniors.

Similar results arise for decreasing discount rates. Our model explains a remarkably large fraction of the hyperbolic discounting patterns found in the data. The parameter γ in Table B.4 captures the hyperbolicity of time preferences. Confidence intervals of this parameter include zero for both groups, indicating that, on aggregate, time preferences do not differ significantly from constant ones. This result empirically confirms that expectations can drive decreasing discount rates *even if* time preference are constant.

To get an idea of how much the change in baseline consumption contributes to decreasing discount rates, we also estimated a restricted-form model not incorporating ex-

[65] Monthly amounts are calculated by x CHF/hour $\times$ 8.5 working hours/day $\times$ 20 working days/month. This calculation assumes that the marginal propensity to consume out of additional income is one. Hence, it can be considered as the lower bound of the expected growth in income.

pectations. In particular, we set c to zero, such that $u(c + x) - u(c) = u(x)$. Time preferences and behavior are then solely characterized by the CRDI discount function. In this model, decreasing discount rates are fully attributed to hyperbolic preferences, captured by the parameter $\gamma > 0$. Such a model, however, is not able to accommodate magnitude-dependency.

As it turns out, the expectations model shows a considerably better goodness of fit compared to the CRDI-model (estimation results for the latter are listed in Appendix B.11, Table B.8). A likelihood ratio test between the two models reveals p-values close to zero for both groups.[66] This is no big surprise, however, as the full model is also capable of explaining behavioral differences in discounting amounts of varying size (magnitude effect).

Departures from exponential discounting (γ) are much more pronounced when c is set to zero. This parameter now lies in the range of 0.7 to 0.8 and is significantly larger than zero. That is, discount rates decline sharply in time horizon and take a clear hyperbolic form. In comparison, for the expectations model we found time preferences not distinguishable from constant ones.

To graphically illustrate the differences in estimated *time preference* parameters of the two models, we rely on the discount rates inferred from η and γ. Figure B.10 plots these functions for both student groups. The dashed curves shows time preferences based on the estimated parameters of the expectations model and the solid curves those of the CRDI-model. The figures depict that our model explains the large part of the decline of discount rates in time horizon. Inferred time preference curves for both student groups become very flat for our model, and, as indicated by the parameter estimates in Table B.4, are not distinguishable from horizontal lines. On aggregate, behavior is therefore reproduced adequately by constant time preferences.

All in all, our structural econometric model explains a larger part of the huge discount rates and their decline in time horizon than we were able to explain with our binary measure for income expectations. We also found indirect evidence for the magnitude effect. The group with higher estimated consumption expectations shows a more pronounced effect.

[66]Estimating the expectations model where γ is set to zero makes no difference here, as γ is not significantly different from zero in the unrestricted original specification. Note that this model also fits better than any hyperbolic discounting model we tested. Results are available on request.

B.4 Discussion and Conclusion

We introduce a novel and unifying explanation for "anomalies" in intertemporal choice and present empirical support for it. In the theoretical part of the paper, we find that subjective expectations can have significant effect on discounting behavior and drive systematic departures from exponential discounting. We discuss one particular situation where such departures naturally occur. Limited access to liquidity can prevent impatient consumers with positive, rational expectations to reach a smooth consumption path. Opting for new alternatives materializing at dates where income is expected to be relatively low can help them to, at least partly, overcome these limitations. Our approach stands in contrast to Loewenstein and Prelec (1992) and others, who do not only criticize the assumptions underlying the exponential discounted utility model, but also argue that consumer's preferences are the ultimate driver of "anomalies". This is not the case in our model. The typical consumer may still comply with the standard assumptions, but the limitations imposed by the environment constrain her scope of action which may lead her to reveal anomalous behavior. Moreover, our approach is not a descriptive one, but, rather, it provides a clear intuition for *why* and *by how much* people depart from exponential discounting.

The results in the empirical part of the paper lend strong support for our hypothesis that subjective expectations govern intertemporal choice behavior. Subjects with positive income expectations and limited access to liquidity reveal much higher discount rates and a sharper decline of these rates in time horizon. The student group with higher estimated expectations also exhibits a more pronounced magnitude effect. An interesting insight is that the majority of empirical studies so far was conducted with relatively poor subjects, i.e. subjects who only hold few liquid assets and typically only have limited borrowing opportunities. Students, for example, are not only exceptionally exposed to such liquidity constraints, but usually also hold substantial positive income expectations. According to our model, we therefore expect them to depart much more strongly from exponential discounting than groups not holding such expectations or not facing such constraints. This should be taken into account when predicting behavior of other groups or recommending policies based on experimental findings.

Liquidity constraints may not be the only explanation for a link between subjective expectations and discounting behavior, however. Empirical evidence reports that people are often too optimistic when it comes to evaluating future life events (Weinstein, 1980, 1987; Armor and Taylor, 2002). People typically overestimate their future earnings (Dominitz, 1998) and their ability to resist future temptations (Nordgren et al., 2009). While such consumers obviously do not have rational expectations, this alternative explanation does not contradict our basic model. Having a too optimistic consumption plan can be sufficient to generate the typical "anomalies", even if no liquidity constraints exist or the consumer is not sufficiently impatient.[67] Such consumers' behavior, however, will never be dynamically consistent.

Our results have strong implications. The fact that *rational planners*, i.e. consumers with positive, rational expectations facing liquidity constraints, and *myopic fools*, i.e. consumers with dynamically inconsistent preferences or too optimistic beliefs, can reveal observationally equivalent behavior poses important challenges for the prediction of behavior and the design of suitable policies. One basic problem is that constant discount rates are no longer the proper criteria to identify rational types. Additional information about subjective expectations and liquidity constraints may therefore be needed to make predictions. Similar problems arise when searching for policies which help irrational consumers to behave more rationally. Interventions not taking into account the different causes underlying behavior, may fail to have the desired effect, be a waste of money or even have detrimental effects for those consumers with rational intentions. Policies geared at undersavings, for example, should only affect those consumers not being aware of what they do, but not those with a rational plan and limited liquidity. Mechanisms distinguishing *rational planners* from *myopic fools* are therefore needed. Possible starting points for the development of such mechanisms are these types' different preferences for commitment and the dynamic consistency of their behavior. The rational consumer, for example, is willing to insure herself against exogenous income shocks, but not against her own future behavior. Conversely, hyperbolic or overoptimistic consumers should be concerned about their own behavior. Whether or not they show a preference for commitment, however, depends on their degree of sophistication (O'Donoghue and Rabin, 1999).

We see a number of directions our work may be extended to. First, our model generates numerous sharp predictions not made by other discounting models. Examples are conditions for which we predict increasing discount rates, magnitude-dependency, the close connection between the "anomalies" or the effect subjective expectations have on behavior. These predictions are testable and make our model falsifiable.

Second, richer expectations (and possibly even liquidity constraints) data can help to

[67]Formally, $c_t > c_0 \geq 0 \; \forall t > 0$ must hold. This does not necessarily require the consumer to have a precise consumption plan in mind. The consumer's beliefs about the future, however, must imply that marginal utility derived from consumption is always larger today compared to tomorrow.

better understand how people's beliefs govern behavior. There are a number of papers proposing methods to elicit subjective expectations (Dominitz and Manski, 1996, 1997; Dominitz, 2001; Manski, 2004). Such procedures can be adopted to account for the magnitude, timing, uncertainty and heterogeneity of subjects' expectations. Embedded in dynamic choice experiments, they can provide additional insights into how expectations are formed and how they drive consumer behavior.

Figure B.9: Positive Expectations and Prospect Arguments

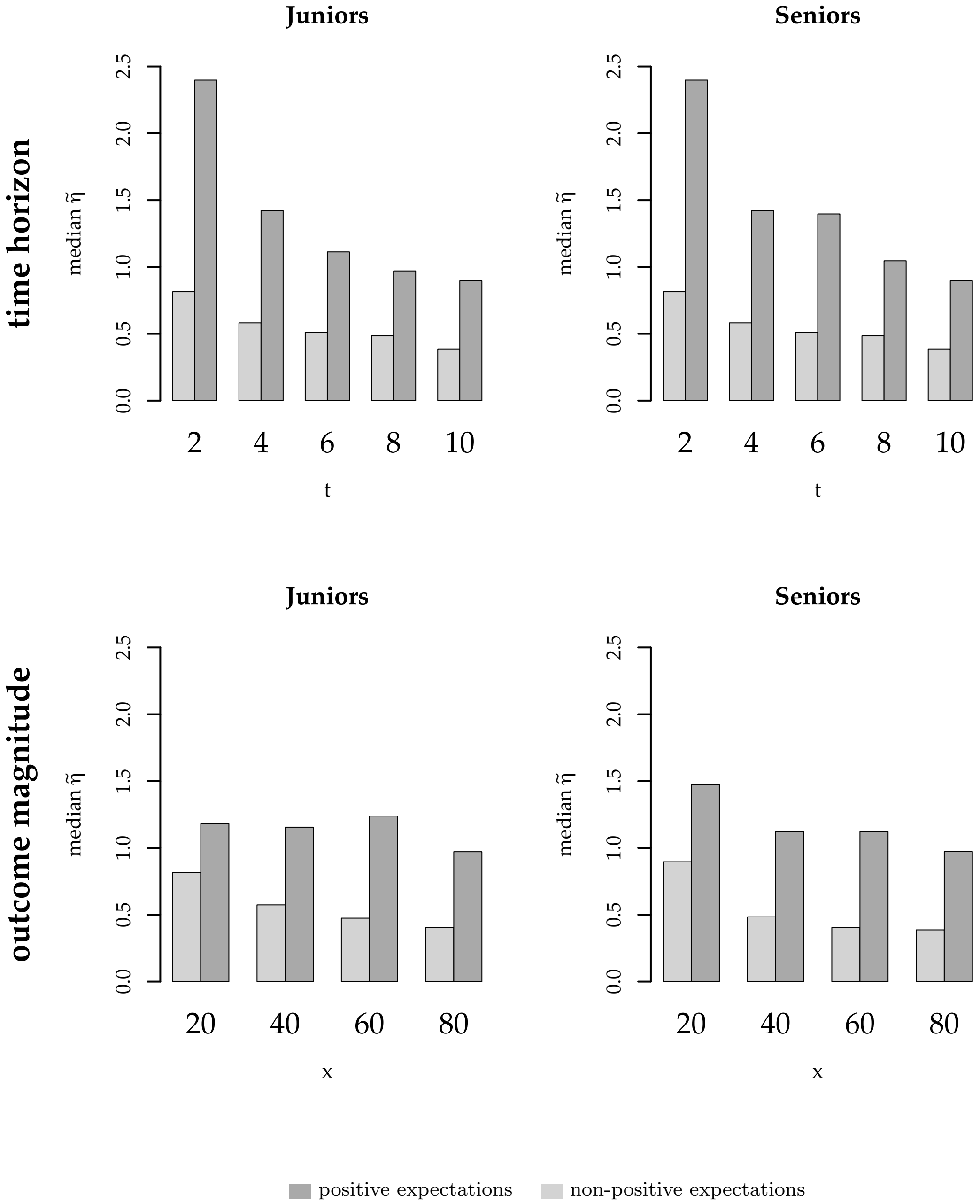
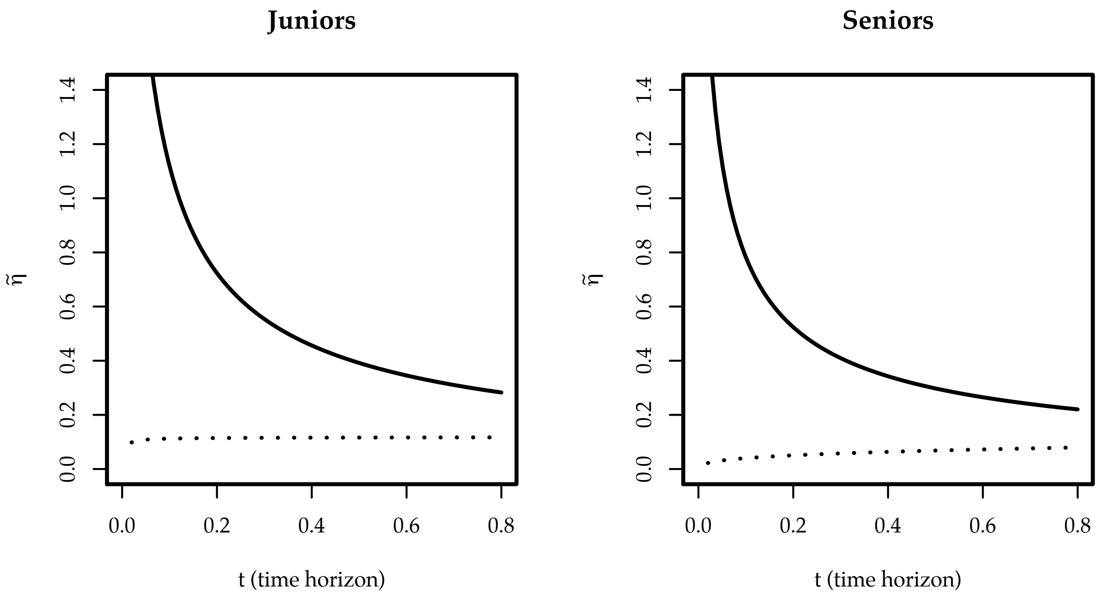


Figure B.10: Inferred Time Preferences



$\cdots$ expectations model — CRDI-model ($c \overset{!}{=} 0$)

B.5 Appendix: Imputed Discount Rates

Our analyses in the theoretical and the empirical part of this paper rely on imputed discount rates. Here we show how these rates are calculated. The basic procedure follows that used in most empirical studies on intertemporal choice (see e.g. Thaler (1981)).

Consider a subject stating her present equivalent y which makes her indifferent to a temporal prospect (x, t). The per annum *imputed discount rate* $\tilde{\eta}$ is then directly inferred from indifference between y and x. That is,

$$y \sim e^{-\tilde{\eta}t} x \,, \tag{B.12}$$

from which it follows that

$$\tilde{\eta} = -\frac{1}{t} \ln\left(\frac{y}{x}\right) \,. \tag{B.13}$$

Intuitively, imputed discount rates reflect the *revealed* propensity to exchange sooner for later consumption. Most empirical studies reject the exponential discounted utility model because these rates are not constant in time horizon and/or depend on outcome magnitude or outcome sign (see for instance Thaler (1981); Benzion et al. (1989); Ahlbrecht and Weber (1995); Chapman (1996) among many others). As our analyses illustrate, however, there are other reasons for subjects to depart from exponential discounting than non-constant time preferences. Subject's imputed discount rates may therefore differ from their rate of time preferences.

To predict behavior, we simply substitute y in Equation B.13 by the present equivalent inferred from our model (see for example Appendix B.6.1).

B.6 Appendix: Anomalies

B.6.1 Time Horizon

According to our model, the present equivalent y making the consumer indifferent to the temporal prospect (x, t) is $y = u^{(-1)} \left[\mathbb{E}\{\delta^t \left(u(c_t + x) - u(c_t)\right) \mid \mathcal{I}_0\} + u(c_0)\right] - c_0$. For the following proofs, we assume that c_t is *a priori* known and that $c_0 = 0$.[68] Note that this consumption plan already contains the consumer's information about liquidity constraints. An expected rise in the consumption plan only takes effect if it occurs prior to the point in time the temporal prospect materializes.

Calculating the discount rate inferred from our model by inserting the predicted present equivalent $y = u^{(-1)}\left[\delta^t\left(u(c_t + x) - u(c_t)\right)\right]$ into Formula B.13, differentiating with respect to t, and reordering, leads to

$$\frac{\partial \tilde{\eta}}{\partial t} = -\frac{1}{t}\left[\underbrace{-\frac{1}{t}\ln\left(\frac{y}{x}\right)}_{\tilde{\eta}} - \underbrace{\left(-\ln(\delta)\right)}_{\eta}\underbrace{\frac{u(y)}{yu'(y)}}_{1/\varepsilon}\right]. \tag{B.14}$$

The interpretation of the term in square brackets is straightforward. It is the difference between the consumer's *behavior*, i.e. her imputed discount rate $\tilde{\eta}$, and her *preferences*, i.e. the product of the her rate of time preference (impatience) η and the reciprocal of the elasticity of her utility function $1/\varepsilon$. The wedge between these two terms is largely governed by how large the consumer expects her consumption to rise during the relevant time horizon. The following comparative statics make that clear.

For u concave and $c_t > 0$, the fraction y/x decreases as c_t increases, ceteris paribus. This is the case since larger c_t's lead to stronger discounting of prospective consumption utility, i.e. due to concavity $u(c_t + x) - u(c_t)$ becomes smaller the larger c_t. As δ^t is a constant (with $0 < \delta^t \leq 1$) and $u^{(-1)}$ is a strictly monotone function, the imputed

[68] The first assumption is purely technical and allows to ignore the expectation operator. Relaxing the second assumption does not affect our basic result that discount rates decline in time horizon for positive expectations and concave instantaneous utility. The fact that new alternatives are not evaluated in isolation, but integrated into existing plans, however, can produce decreasing discount rates by itself without requiring increasing baseline consumption. For u concave and $\delta < 1$ (impatience), for instance, discount rates decline if $c_0 = c_t = c > 0$. Even if u is isoelastic, $u(c_t + x)$ is not if $c > 0$ is included into the function for the same reason described in Wakker (2008). Distortions, i.e. departures from constant imputed discount rates, are then produced by positive rates of time preference. Although this effect has the same sign as the one produced by positive expectations, it seems very unlikely that the empirical findings are driven solely by it. There are two reasons for that. First, if this would be the case, we should not find a correlation between self-reported income expectations and decreasing discount rates. We do, however, find such a relationship in the empirical part of the paper. Second, there are no plausible parameter combinations fitting the hyperbolicity of behavior in our data. A calibration exercise shows that behavior revealed by the two student groups in the empirical part of this paper are only consistent with baseline consumption levels of CHF 25.50 (juniors) or CHF 35.90 (seniors), respectively. Interpreting these amounts as per-hour consumption, as done in Section B.3.3, leads to baseline consumption levels of CHF 4335 to CHF 6103 per month, i.e. consumption levels that are five to over eight times larger than the average subjects monthly income. The calculations are based on the assumptions described in Footnote 65.

discount rate $\tilde{\eta}$ increases as consumption expectations c_t grow larger. If u is isoelastic, $1/\varepsilon$ is constant. Under our assumptions, the difference in square brackets is positive and $\partial\tilde{\eta}/\partial t < 0$. Hence, imputed discount rates decline in time horizon, with higher consumption expectations leading to a more pronounced effect. □

A number of additional findings result from Equation B.14. First, (expected) baseline consumption changes do not take effect under utility linear in consumption, simply because c_t cancels out in $\tilde{\eta}$ and $1/\varepsilon = 1$. The same is the case for $c_t = 0$ if u is isoelastic. Second, in the long run, imputed discount rates are not distinguishable from constant rates even if $c_t > 0$, since

$$\lim_{t\to\infty} \frac{\partial\tilde{\eta}}{\partial t} = 0\,. \tag{B.15}$$

Third, contrary to hyperbolic discounting models, decreasing discount rates are not "hardwired" in the consumer's preferences, but are the result of the consumer's expectations and the borrowing limitations preventing her from smoothing away the preceding low-consumption periods. The size of the effect therefore depends on c_t in our model, but there should be no such dependence in hyperbolic preference models not incorporating these drivers of behavior. Fourth, it directly follows from Equation B.14 that there is an interaction between decreasing discount rates and the magnitude effect. Holding c_t constant, imputed discount rates decline less strongly in time horizon the larger x becomes, ceteris paribus (see Appendix B.6.2).

B.6.2 Outcome Magnitude

The derivative of the imputed discount rates with respect to the outcome magnitude is

$$\frac{\partial\tilde{\eta}}{\partial x} = -\,\frac{1}{t}\left[\underbrace{\frac{\delta^t u'(c_t+x)}{u'(y)}}_{\mathrm{MRS}_{c_t,c_0}}\frac{1}{y} - \frac{1}{x}\right] \tag{B.16}$$

where $y = u^{(-1)}\left[\delta^t\left(u(c_t+x) - u(c_t)\right)\right]$.[69] The size of the magnitude effect largely

[69]We make the same assumption as in Appendix B.6.1 to eliminate the expectation operator. Due to the same reasons described in Appendix B.6.1, we prove the existence of the magnitude effect for the case where the stationary component of baseline consumption is zero. Positive, but constant baseline consumption levels can induce an opposite magnitude effect. There is, however, no reasonable parameter

depends on how much the consumer expects her consumption to rise during the relevant time horizon. The first term in square brackets, the product of the marginal rate of intertemporal substitution between future and present consumption MRS_{c_t,c_0} and $1/y$, rises as c_t becomes larger.

The following comparative static proves the magnitude effect and its dependence on c_t. Consider two outcomes with $\bar{x} > \underline{x} > 0$. $u' > 0$ implies that $y_{\bar{x}} > y_{\underline{x}}$, ceteris paribus. From $u'' < 0$ it follows that $\left(u^{(-1)}\right)'' > 0$ and $y'' > 0$. As a result, the discount fraction $y_{\bar{x}}/\bar{x}$ is larger than the discount fraction $y_{\underline{x}}/\underline{x}$. Since $\tilde{\eta}_{\bar{x}} < \tilde{\eta}_{\underline{x}}$, imputed discount rates decrease as x grows larger. Using the same procedure it can easily be shown that the effect is more pronounced as c_t increases, holding x and all other things fixed. □

Some additional findings result from Equation B.16. First, (expected) changes in baseline consumption do not take effect if $u(z) = z$, i.e. if $u'(z) = 1$. For this case, no magnitude effect is predicted. Similar results are obtained if $c_t = 0$ and u isoelastic. Then, the first term in square brackets reduces to $1/x$, which is the lower bound of this product given that $c_t \geq 0$ and u concave. Second, imputed discount rates converge to rates constant in outcome magnitude as x grows to infinity, holding all other things fixed, as

$$\lim_{x\to\infty} \frac{\partial \tilde{\eta}}{\partial x} = 0\,. \tag{B.17}$$

Third, the magnitude effect diminishes as the time horizon tends to infinity, since

$$\lim_{t\to\infty} \frac{\partial \tilde{\eta}}{\partial x} = 0\,. \tag{B.18}$$

Once again, this result indicates that the two effects, diminishing impatience and the magnitude effect, are closely intertwined.

combination which fits the magnitude effect in our data, given that u is concave, $1 \geq \delta > 0$ and $c_0 = c_t = c > 0$.

B.7 Appendix: Dynamically Consistent Hyperbolic Discounting

Hyperbolic discounting can be dynamically consistent if the consumer holds rational expectations, such that her expectations do not differ systematically when reevaluating the same choice at some later point in time.

Dynamic consistency, or stationarity, implies that no preference reversals occur if outcomes are shifted by a common delay (Fishburn and Rubinstein (1982)).

Definition 1 (Stationarity) *If* $(x, t) \sim (y, t + \lambda)$ *then* $(x, s) \sim (y, s + \lambda)$,

for all outcomes $\{x, y\}$, all dates $\{t, s, t + \lambda, s + \lambda\}$ and $\lambda > 0$.

Consider a consumer who is indifferent between the two temporal prospects (x, t) and $(y, t + \lambda)$ when evaluating them now. At this point in time she expects her baseline consumption to be c_t for t *and* $t + \lambda$.[70] For stationarity to hold, indifference must still be established when evaluating the same two prospects in $t - s$ periods from now. New information about future consumption becoming available in the meantime, however, may lead the consumer to update her expectations such that $c_s = c_t + \mu$.

Formally, our model implies that the following indifference condition holds when the two temporal prospects are evaluated now:

$$\delta^t \left[u(c_t + x) - u(c_t)\right] = \delta^{t+\lambda} \left[u(c_t + y) - u(c_t)\right] . \tag{B.19}$$

Rational expectations imply that μ is independent and identically distributed with expected value of zero. In this case, the consumer is still indifferent between the two prospects, when evaluating them in $t - s$ periods, as the following holds:

$$\delta^s[u(c_t + \mu + x) - u(c_t + \mu)] = \delta^{s+\lambda}[u(c_t + \mu + y) - u(c_t + \mu)] . \tag{B.20}$$

In other words, discounting does only depend on the delay λ and the consumer will not reverse preferences. □

[70] For expositional simplicity, we leave the expectation operator aside.

B.8 Appendix: Consumption Spendings

Table B.5: Planned Appropriation of Payouts

Purpose	Tomorrow	Later
consumption$^{(1)}$	94.55% (104)$^{(2)}$	76.36% (84)
savings$^{(3)}$	3.64% (4)	6.36% (7)
debt repayments	1.82% (2)	0.91% (1)
spend before received	-	2.73% (3)
no specific plans	0.00% (0)	13.64% (15)

[1] Consists of daily consumption spendings, installments against consumer durables and gift purchases. We also asked subjects whether they plan to donate the payouts. No single subject did so.
[2] Number of subjects in parentheses (total: 110 subjects).
[3] None of the 110 planned to use the payout for capital investment.

B.9 Appendix: Descriptive Results for Risk Data

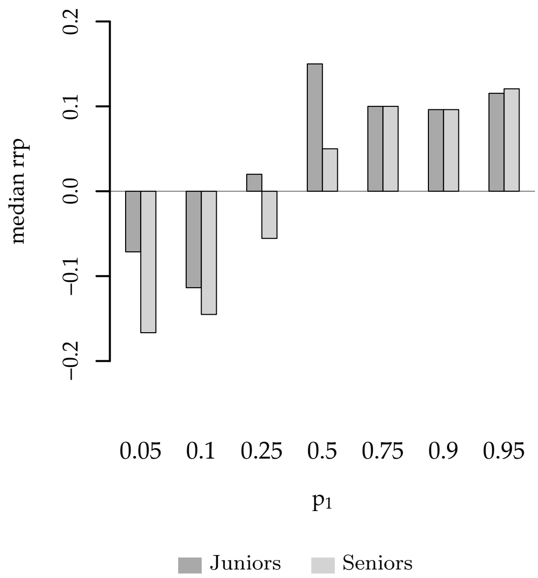
Figure B.11 shows median relative risk premia for the probabilities subjects were confronted with. This measure is defined as

$$\mathrm{rrp}(\mathcal{R}) = \frac{\mathrm{ev}(\mathcal{R}) - \mathrm{ce}(\mathcal{R})}{|\mathrm{ev}(\mathcal{R})|}, \tag{B.21}$$

where $\mathrm{ev}(\mathcal{R})$ denotes the expected value of the lottery $\mathcal{R}$ and $\mathrm{ce}(\mathcal{R})$ the observed certainty equivalent. $rrp > 0$ ($rrp < 0$) represents risk aversion (proneness), and $rrp = 0$ risk neutrality.

As depicted by the figure, risk aversion strongly depends on the probability under consideration. On aggregate, subjects are risk-seeking for low-probable gains, but risk-averse for medium- and high-probably gains. This motivates the use of a inverted S-shaped probability weighting function as we do in our econometric model.

Figure B.11: Relative Risk Premia



B.10 Appendix: Income Expectations

Table B.6: Summary Statistics

	Juniors		**Seniors**	
	Mean	Std.Err.	Mean	Std.Err.
Positive Expectations	0.301	0.014	0.260	0.014
Income	525.840	11.378	1231.000	31.072
Investment Experience	0.105	0.009	0.320	0.015
Female	0.501	0.015	0.500	0.016
CRT Score	0.561	0.011	0.647	0.010
Observations(1)	1101		1000	

1 omitting subjects with missing observations in income and CRT

Table B.7: OLS Regressions

Dependent Variable: $\mathrm{med}(\tilde{\eta}_i)^{(1)}$

	Juniors		Seniors	
Coefficient	I	II	I	II
Intercept	0.672***	0.100	0.686***	0.853
	(0.090)	(0.825)	(0.102)	(0.645)
Positive Expectations$^{(2)}$	0.827***	0.910***	0.576***	0.611***
	(0.287)	(0.315)	(0.200)	(0.215)
Log-Income$^{(3)}$		0.144		0.004
		(0.140)		(0.083)
Investment Experience		-0.137		0.035
		(0.403)		(0.229)
Female		-0.189		0.032
		(0.218)		(0.199)
CRT Score		-0.370		-0.355
		(0.285)		(0.312)
R^2	0.195	0.246	0.142	0.172
Observations$^{(4)}$	56	56	50	50

*** indicates significance on the 1% level; standard errors (in parentheses) are based on the bootstrap method (999 replications)

1 calculated by taking the median over all observations of subject i

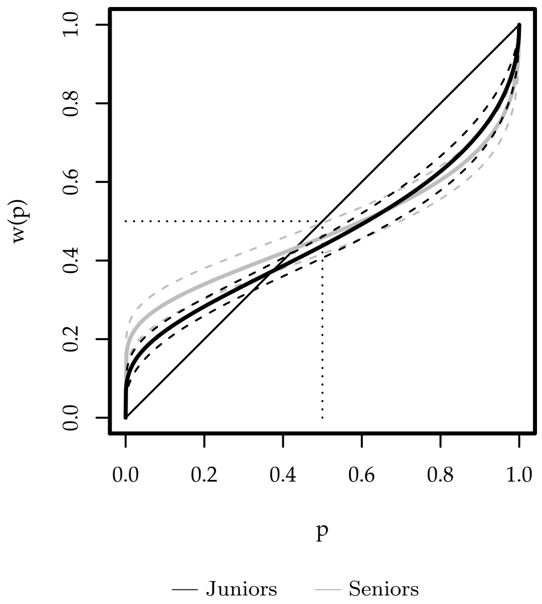
2 summary statistics for all variables are listed in Table B.6; coefficients are robust when restricting on subsamples of the data, such as gender

3 in CHF; the exact transformation is log(income+1) as there were subjects with zero income

4 subjects with missing observations in income and the CRT score are omitted

B.11 Appendix: Additional Estimation Results

Figure B.12: Probability Weighting Functions



dashed curves are 95% confidence
bands based on percentile bootstrap;
probabilistic risk neutrality holds
if $w(p) = p$; dotted lines mark 0.5

Table B.8: Estimation Results - CRDI-Model

	Juniors			**Seniors**		
	$p.e.$	$q_{0.025}$	$q_{0.975}$	$p.e.$	$q_{0.025}$	$q_{0.975}$
η	0.345	0.130	0.894	0.235	0.173	0.742
γ	0.795	0.536	0.963	0.719	0.505	0.940
ρ	0.237	0.172	0.300	0.188	0.111	0.251
α	0.497	0.419	0.575	0.614	0.556	0.670
β	0.007	-0.065	0.059	0.098	-0.013	0.189
$\ln \mathcal{L}$	-7206			-6832		
BIC	15247			14438		
Subjects	57			53		
Observations	1117			1060		
Parameters	119			111		

95% confidence intervals are derived by the percentile bootstrap method with 9999 replications. The resampling procedure accounts for the panel structure of the data (clusters). Parameters include additional estimates for $\hat{\xi}_i$ for domain- and individual-specific error variances.

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Appendix C

Preferences or Constraints? A Rational Explanation for Probability-Dependent Risk Attitudes

This chapter has not yet been published elsewhere.

C.1 Introduction

A large body of empirical evidence, accumulated over the last half century, shows that despite its normative appeal expected utility theory is not suited well as a descriptive theory of choice under risk. None of its key premises has been challenged more severely than the postulate that preferences are linear in probabilities.[1] Empirical evidence suggests that humans and animals alike systematically violate this assumption (Allais (1953), Kahneman and Tversky (1979), Battalio et al. (1985), Kagel et al. (1990), among others). They tend to overweight small-probability outcomes and underweight large-probability outcomes. This finding compromises expected utility theory at its core. It appears that risk taking behavior can be characterized only inadequately by the curvature of the utility function. Instead, there seem to be other important factors shaping behavior.

Richer models of choice under risk have been proposed, the most popular class being rank-dependent utility (RDU) models (Quiggin, 1982; Luce and Fishburn, 1991; Tversky and Kahneman, 1992).[2] RDU models preserve many of the normatively appealing properties of expected utility, but are less restrictive about how preferences depend on probabilities.[3] As nonlinear weighting of probabilities seems to be a pervasive feature of individual risk taking behavior (see for instance Bruhin et al. (2010)), RDU models turn out to be a useful descriptive generalization of expected utility.

The availability of better descriptive theories of choice under risk, however, is only one side of the coin.[4] For most applications it is inevitable to understand *what* actually drives risk taking behavior. The design of appropriate incentive schemes, for example, often requires the policy maker to know the reasons underlying behavior. In particular, if the policy maker aims at preventing individuals from taking excessive risks in a particular situation, she must know whether such behavior is due to errors, trait or rational reasons.[5] Only with this knowledge is it possible to implement policies which affect the right people in the right way. This is of particular relevance for policies obeying the principle of asymmetric paternalism (Camerer et al., 2003). This principle says that corrective interventions geared towards particular behaviors should only have an impact on those

[1]The independence axiom states that if the decision maker prefers ($\succ$) a lottery A over a lottery B, i.e. $A \succ B$, then $pA + (1-p)C \succ pB + (1-p)C$ for all lotteries C and all probabilities $p \in (0, 1)$ must hold. The axiom restricts the functional form of the model to be linear in probabilities (see Machina (1982) for a more elaborate discussion on this).

[2]See Gilboa (1987) and Schmeidler (1989) for similar theories of choice under uncertainty.

[3]Quiggin (1993) provides an extensive discussion on this topic.

[4]See also Starmer (2000).

[5]A more specific and contemporary example are incentive schemes which aim at preventing fund managers from investing in too risky assets.

economic agents who behave in an irrational way.[6] If policy interventions affect rational agents or are based on incorrect assumptions about preferences, they may be a pure waste of money, or even worse, have adverse welfare effects.

The objective of this paper is to demonstrate that apparent probability distortions can be the result of rational choice. We argue that the carriers of risk taking behavior are not only individual preferences, but also restrictions imposed by the environment, and present the following main insights.

First, it is easy to find situations in which the environment can be identified as a predominant driver of risk taking behavior. We give three examples that underscore the wide economic relevance of such situations and their prevalence in various choice domains. Without needing ancillary assumptions about the utility function, these examples illustrate how environmental factors can force decision makers to undertake excessive risks.

Example 1 (Foraging) *Imagine a bird in an environment which does not provide much food (e.g. a cold winter). The bird only has very little energy reserves left and, hence, needs to find some food to survive the upcoming night. In its struggle for survival, it is confronted with two foraging alternatives: A) either picking the few remaining grains from a feeding place, or B) picking a larger quantity of grain from the ground where a cat is lurking. If the few grains in the feeding place preclude reaching the minimal subsistence level, i.e. if choosing option A leads to certain death, it must opt for the risky option B, although there is some likelihood of getting eaten by the cat. Only the risky choice involves the possibility of survival.*

Example 2 (Consumption) *Consider a liquidity-constrained consumer, i.e. a consumer who only holds few liquid assets and is not permitted to borrow money. The consumer has to meet her existential needs (e.g. to consume food), has to honor her contracts (e.g. to pay rent) and has to settle her bills. Assume that she has the choice between two investments: A) a certain one leading to a small interest payment (bank), and B) a risky one leading either to a large gain or nothing (stock). For simplicity, assume that the outcomes of these investments are paid out immediately after making the choice. If the outcome from option A together with the consumer's liquid assets does not allow her to satisfy her needs, she should opt for option B. This option gives her the chance of escaping from getting prosecuted or starving.*

Example 3 (Firm) *Assume a firm is running out of liquidity. The management must realize sufficient profits in order to avoid Chapter 11 bankruptcy. It has the choice to*

[6]This excludes the case where rational behavior has negative external effects.

invest in one of two projects: A) a project generating only small profits, but for sure, and B) a risky project generating large profits if it succeeds, but none if it fails. If paying off sufficient debts to avoid bankruptcy or significant costs of distress is not possible with the profits generated by project A, the management should better opt for project B. Only this project allows the company to avert imminent bankruptcy.[7]

All these examples have a number of features in common. The decision maker must achieve a certain, possibly exogenously given, threshold *(minimal energy level required to survive or sufficient profit to avoid bankruptcy)*. We refer to it as the *minimal subsistence level*. Not reaching it leads to direct negative consequences *(death, prosecution or bankruptcy)*, so-called *costs of distress*. The commodity the decision maker requires *(food or liquidity)*, however, is scarce in that she only possesses a small quantity of it and only has limited (or no) market access. This latter restriction prevents her from trading another commodity (e.g. money) in exchange to additional units of the commodity of interest, and from intertemporally reallocating units of the commodity in order to have more of it at the disposal today. Therefore, to reach the minimal subsistence level, the decision maker has no other option but choosing the more risky option B. Only with this choice, she may attain a *terminal outcome*, i.e. the assets at her disposal plus the outcome from the option she has chosen, lying above the required level. These examples therefore illustrate that the pressure imposed by the environment can have decisive impact on behavior.

Second, we derive a simple model capturing this intuition. We show that risk taking behavior can be adequately modeled using standard expected utility, but taking into account the negative consequences materializing when the minimal subsistence level is not achieved. Being exposed to constraints may lead even rational expected utility maximizers to reveal risk attitudes which naturally depend on probabilities. It is their response to the environment which generates systematic departures from standard behavior. Our model produces a wide range of predictions beyond probability-dependent risk attitudes which dovetail nicely with observed risk taking behavior. Among others, these concern common ratio violations, stake effects, the heterogeneity in risk taking behavior and choice domain dependent risk attitudes. Probably most interestingly, most of the patterns we predict are usually explicitly listed as evidence against expected utility theory. Put differently, our approach constructs the theoretical bridge between the standard preference model

[7] Consistent with this example, Bowman (1980, 1982) finds that managers of troubled firms take risks that they would not take in other situations. Similar findings are reported for farmers (Kunreuther et al., 1979).

and effectively observed behavior.

Third, our findings have material implications. We already mentioned that the design of appropriate policies commonly requires to know the actual drivers of behavior. This paper provides one possible answer to that question and stresses the importance of developing mechanisms which allow to distinguish between different causes of probability distortions. Apparent violations of expected utility theory, and especially violations of the assumptions that preferences are linear in probabilities (i.e. the independence axiom), may be due to environmental constraints rather than to a failure of the standard preference model. This is an important finding. Preferences, as a central primitive of economic behavior, should be stable across different choice domains and over time.[8] Evidence suggest that they are not (e.g. Deck et al. (2009) and Zeisberger et al. (2010)). This issue may be resolved by taking into account exogenous factors affecting choice behavior. In fact, our model shows that it is possible to accommodate the most important behavioral patterns of choice under risk without discarding or needing to generalize expected utility. While both, our model and models incorporating probability weighting capture similar properties of risk taking behavior, there are many situations for which their predictions differ in fundamental ways. For example, our model predicts that risk taking behavior depends on both, stake size and choice domain. Decision makers are predicted to depart less strongly from linear probability weighting as stakes grow larger or the commodity of interest in the respective choice domain becomes less scarce, ceteris paribus. Facilitating market access may be one possible policy intervention encouraging economic agents to act in a way closer to their true preferences. In contrast, RDU models do neither make such predictions, nor do they point at solutions for such problems. They only capture revealed preferences and not the mechanism generating behavior. RDU parameter estimates stemming from one study should therefore only be used with care when making predictions or calibrating models for different choice domains, different subjects or different points in time.

The remainder of this paper is organized as follows. Section C.2 reviews the related literature. Section C.3 derives a simple model of choice under risk and fleshes out its basic properties leading to departures from standard expected utility. Section C.4 presents the predictions the model makes in a constrained environment and compares these predictions to settings where no such limitations apply. Section C.5 summarizes the most central results, outlines their implications, and concludes.

[8] We abstract from changing tastes.

C.2 Related Literature

This paper is related to previous theoretical and empirical research focussed on departures from standard expected utility behavior. Here, we briefly review the - in our opinion - most relevant theoretical contributions. Specifically, we compare our approach to rank-dependent probability weighting and other models which (can) induce nonlinearities in probabilities. References to empirical studies can be found in Section C.4.

Our approach compares to RDU evaluation (Quiggin, 1982). Rank-dependent weighting posits that cumulative probabilities are transformed into decision weights.[9] Under this weighting scheme, the decision maker ranks all outcomes in the probability distribution with respect to their size and attaches to each outcome utility the corresponding weight, where the sum of all weights is equal to one. The decision weight associated with a particular outcome remains the same as long as the ranking of this outcome relative to all other outcomes in the distribution remains the same, but it can change abruptly when the rank order of the outcomes changes. While sharing some characteristics with this weighting scheme, our approach differs in several important aspects. On the level of preferences, we still retain linearity in probabilities, i.e. the independence axiom. However, as terminal outcomes below a certain minimal subsistence level lead to costs - costs which the decision maker integrates - behavior may depart from standard expected utility. The imputed weighting scheme is dichotomous in the sense that it is not the original rank of an outcome in the distribution, but the position of the terminal outcome relative to the minimal subsistence level which governs its attractiveness. The ties between outcome and probability evaluation are therefore much stronger as compared to RDU.[10] As a consequence, the predictions our model makes differ in some parts substantially from those made by RDU models. We discuss some examples in Section C.4.

Mainly motivated by the co-existence of gambling and taking out insurance, nonlinear weighting of probabilities has become a central ingredient of the most popular descriptive theories of choice under risk. Numerous explanations have been proposed for why people distort probabilities.[11] In particular, it has been suggested that perceptional, motivational or emotional processes, or an interaction between these processes, drive expected utility

[9]Earlier approaches used direct transformation of probabilities into decision weights (Handa (1977); Karmarkar (1978); Kahneman and Tversky (1979), among others). This may lead to violations of dominance.

[10]There exists empirical evidence showing that the dependency between the evaluation of outcomes and probabilities is only inadequately captured by RDU models (Fehr-Duda et al., 2010).

[11]Not all papers cited in the following do explicitly discuss probability weighting. However, all these theories can, in principle, produce patterns similar to probability distortions.

violations.

Perceptional and psychophysical explanations for overweighting of small probabilities and underweighting of large probabilities have been brought forth by psychologists. Tversky and Kahneman (1992), for instance, argue that subjects perceive certainty and impossibility as natural reference points on the probability scale.[12] Subjects become less sensitive to changes further away from these reference points, i.e. the probabilities zero and one.[13] This *diminishing sensitivity* implies that the probability weighting function is steepest near the endpoints, but becomes flatter as probabilities become more remote from the extremes. Inverted S-shaped probability transformations capture this characteristic. Taking up this reasoning, Gonzalez and Wu (1999) allude that the probability weighting function "characterizes the psychophysics of chance" (p.135). Due to this interpretation, departures from linear probability weighting are often attributed to optimism or pessimism (see for instance Quiggin (1982); Yaari (1987)).

Another strand of research claims that motivational reasons drive departures from expected utility theory. Lopes (1986) argues that "weights reflect individuals' goals and not their perception of probabilities or values" (p.276). Her theory, labeled security-potential/aspiration (SP/A) theory, encompasses "two logically and psychologically separate criteria" (Lopes and Oden (1999), p.290). First, the security-potential (SP) criterion which is represented by a probability transformation function. This criterion captures the decision maker's predisposition to risks. Motivated by fear or hope, decision makers are either security-minded (risk averse) or potential-minded (risk seeking). Second, the aspiration (A) criterion which governs the attractiveness of certain lotteries given the decision maker's goals. These two separate criteria conjointly determine the decision maker's choice. However, Lopes argues that "strategies that involve maximizing the probability of meeting a goal or aspiration level are fundamentally different from strategies of maximizing expected utility" (Lopes (1986), p.277).

The differences to our approach are twofold. First, we integrate expected utility maximization and environmental pressure into one coherent maximization problem and, unlike Lopes, allow for utility nonlinear in outcomes. In our model, the costs of not meeting the minimal subsistence level go directly into the utility function. Second, in our approach probability distortions emerge endogenously from the decision maker's response to environmental constraints. They are inextricably linked to the minimal subsistence level she

[12]See Tversky and Kahneman (1986) for a related discussion.

[13]The gap between objective and subjectively perceived intensities to stimuli was already discussed by Weber (1834) and Fechner (1860).

has to attain, and are not a separate criterion.

In the context of configural-weight theory, Birnbaum et al. (1992) present a different motivational explanation. They point out that over- and underestimation of probabilities, i.e. errors or deviations, lead to asymmetric negative consequences. The decision maker anticipates these consequences and adjusts her behavior by minimizing them.[14] This approach shares some similarities with regret theory (Bell, 1982; Loomes and Sugden, 1982).[15] As its name implies, regret theory conjectures that the decision maker experiences regret when uncertainty is resolved because she ponders on how much better the outcome would have been had she chosen differently.

Also very closely related to these approaches are disappointment aversion models (Bell, 1985; Loomes and Sugden, 1986; Gul, 1991; Walther, 2003). Similar to the motivational explanations, the decision maker expects a specific outcome to materialize. As actual outcomes, however, may differ from expected outcomes, the decision maker experiences disappointment if the actual outcome lies below her expectations or experience elation if it lies above her expectations. These emotions are anticipated at the decision stage.[16]

Our explanation has the following distinguishing features when compared to explanations involving motivational processes (in particular Birnbaum et al. (1992)) or anticipatory feelings (regret and disappointment theories). First, it is not the expected outcome which separates elation from disappointment outcomes or higher from lower psychological "costs", but an exogenous minimal subsistence level entailing monetary costs only in the case the terminal outcome falls below it. Said differently, it is not a subjective reference point formed by endogenous expectations discriminating gain from loss outcomes, but the final state relative to an objectively given, exogenous threshold level which builds the core of our model. Second, we primarily see the consequences as monetary costs of quantifiable size. The problem with integrating psychological or emotional "costs" is that the existence and size of such consequences is hardly objectively verifiable. Consequently, such approaches may be used to rationalize almost any behavior. Third, contrary to

[14]An example may be a cook who must decide how much chicken soup to prepare for his guests. When cooking too much he wastes food (and money), when cooking too little his guests will not have enough. Similar reasons are discussed by Hogarth and Einhorn (1990), Weber (1994), and Weber and Kirsner (1997). Hogarth and Einhorn (1990) argue that "cognitive and motivational factors [...] both play important roles in determining decision weights" (p.780). Weber (1994) examines the Birnbaum argument in a detailed manner. Weber and Kirsner (1997) conduct an experiment and apply a number of manipulations in order to test different reasons for RDU evaluation. They find partial support for both, the Birnbaum hypothesis and perceptual biases.

[15]Preferences in this class of models are not necessarily transitive.

[16]It may be argued that departures from expected utility due to anticipated regret or disappointment aversion are rational. Such a view is defended by Loomes and Sugden (1982) (regret theory) and Loomes and Sugden (1986) (disappointment aversion theory).

most models above, these costs are of an absolute nature and only affect terminal outcomes smaller than the minimal subsistence level. That is, there is no additional function penalizing and rewarding outcomes relative to a certain reference level.

Another explanation that has been proposed for probability weighting are dual process or dual self models. An example is Loewenstein and O'Donoghue (2007). They argue that probability weighing is the result of an interplay between emotional and cognitive processes. The difference to our approach is evident: Our model is a *single process* (and *single self*) model. As such it is much more tractable, has testable implications and makes more precise, verifiable predictions.

There are some interesting parallels between our work and risk-sensitive optimal foraging theory (see McNamara and Houston (1992) for a review). This branch of the literature is concerned with optimal behavior in risky and changing environments (see e.g. McNamara (1996)). McDermott et al. (2008) use such a model to show that the S-shaped value function of prospect theory may have resulted from evolutionary adaption, i.e. our ancestors' adjustment to their environment. Our model can be used to develop a similar foundation for nonlinear weighting of probabilities.[17] We motivate such an approach in a later part of this paper.

C.3 Model

In this section we obtain and motivate a simple model of choice under risk. Our main interest thereby lies on how the environment can force an expected utility maximizer to deviate from standard behavior. The section starts with a discussion of our modeling assumptions. They consist of three parts: the decision maker's preferences, the limitations imposed by her environment and the actual choice problem. An illustration of the mechanism generating apparent expected utility violations closes this part of the paper.

C.3.1 Preferences

Our decision maker obeys the von Neumann and Morgenstern (1947) (vNM) axioms. Her preferences are characterized solely by a utility function u. u satisfies the common assumptions imposed on utility functions, i.e. it is globally differentiable and strictly increasing in outcomes. If not stated otherwise, we assume that the function is concave,

[17]It would be interesting to see how these two approaches integrate into one unifying model. A discussion of this point, however, is beyond the scope of this paper and we leave it up to future research to answer this question.

i.e. that the decision maker is risk averse. On the level of preferences all attractive properties of expected utility theory are therefore retained, the most prominent one being the linearity of utility in probabilities.

Consistent with expected utility theory, we presume asset integration.[18] The decision maker evaluates all lottery outcomes by first integrating them with her *assets* ω. Accordingly, we define u over *terminal outcomes* $\omega + x$. If not stated otherwise, we consider the decision maker's assets as part of her environment as it turns out to be a crucial determinant of whether or not she is restricted with respect to her behavior.

C.3.2 Environment

The decision maker's environment is made up of three components: a constraint limiting her scope of action, an assumption which ensures that the constraint is binding, and the consequences she faces if the terminal outcome falls below the minimal subsistence level. In what follows, we motivate these three components in detail.

First, the constraint in our model takes the form $\omega \geq 0$, i.e. the decision maker holds a zero or positive quantity of assets ω.[19] This assumption ensures that the decision maker only has at her disposal the quantity of the commodity in question she accumulated over past periods. Put differently, it prevents her from borrowing additional units of the commodity or to trade quantities of a second commodity in for additional units of the commodity. Such limitations apply if either the commodity is not tradable, i.e. if no corresponding market exists, or if the decision maker's access to such a market is restricted. We further assume that the decision maker is relatively poor with respect to the commodity, i.e. that her assets ω are relatively small.

Second, obviously, such constraints are not binding if there exists no immediate necessity to obtain additional units of the commodity. We ensure this by introducing the *minimal subsistence level* τ. The decision maker is required to reach a terminal outcome which satisfies this threshold level. We shall assume that she cannot achieve this goal by her assets alone.

Third, if the decision maker does not reach the minimal subsistence level, she faces some immediate and detrimental consequence. We model this consequence as a cost and refer to it as *cost of distress* κ. To keep our analysis concise, we assume that the actual

[18] Although von Neumann and Morgenstern (1947) do not explicitly state this assumption, Wakker (2005) defends this position by pointing out that behavior can only be considered as rational if outcomes are defined in terms of final wealth.

[19] Deaton (1991) proposes a buffer-stock model of consumption-saving behavior and makes similar assumptions.

consequence is known in advance and that the corresponding cost is deterministic.[20] It becomes immediately apparent what role the assets ω play in this context. Holding all other things fixed, the likelihood of falling below the minimal subsistence level decreases as the decision maker's assets grow larger. Decision makers who are relatively poor with respect to the commodity in question are therefore more likely to bear the negative consequences associated with low terminal outcomes. As a result, they should be more susceptible to departures from standard behavior. We come back to this conjecture later.

In the examples presented in the introduction, the environment is characterized as follows. It is the limited energy reserve *(foraging example)* and the limited access to liquidity *(consumption and firm example)* preventing the decision maker from choosing according to her true preferences. In all situations, the two crucial requirements are met: First, the decision maker is not endowed with a sufficient quantity of the commodity, and, second, there is no possibility to circumvent this restriction by accessing a market. The constraint is binding because in all cases there exists a minimal subsistence level larger than the decision maker's assets. The bird in the foraging example needs to attain a sufficiently large energy level to survive. The consumer has to meet her existential needs, e.g. by purchasing food, and has to comply with her contracts, e.g. paying her rent. The firm has to realize enough profit to pay off its debts and avoid Chapter 11 bankruptcy. Not accomplishing these goals has direct consequences. The bird may die from starvation, the consumer may be prosecuted because she does not fulfill her contracts, and the firm may go bankrupt.

C.3.3 Choice and Maximization Problem

We consider the choice among a finite set of simple lotteries $\mathcal{L}$, i.e. probability distributions on the outcome space X. X is defined over terminal outcomes and has finite support, i.e. $\exists \{\omega + \underline{x}, \omega + \overline{x}\}$ with $\omega + \overline{x} > \omega + \underline{x}$ such that $X = [\omega + \underline{x}, \omega + \overline{x}]$. In other words, the outcome space is delimited by the worst possible terminal outcome $\omega + \underline{x}$ and the best possible terminal outcome $\omega + \overline{x}$ in the choice set $\mathcal{L}$. In our introductory examples, the elements in the choice set are foraging places *(foraging example)*, investment alternatives *(consumption example)* or projects *(firm example)*, where, typically, the decision maker is uncertain about the final outcome of an option at the point in time the decision is made.

Unless stated otherwise, we take probabilities as objectively given. Our approach,

[20] A decision maker may be uncertain about the size of the actual cost materializing when the minimal subsistence level is not reached. In this case, κ is the expected value of the probability distribution over all possible consequences. We do not discuss this refinement in this paper.

however, extends to situations where this is not the case (see Savage (1954)). To keep things simple, we restrict our attention to atemporal settings. That is, the environment remains unchanged between the point in time the decision is made and the point in time uncertainty is resolved. During this time, the decision maker is not allowed to consume out of her assets. In this sense, the situation we consider is very similar to the typical setting usually implemented in risky choice experiments.[21]

In the situation considered here, the decision maker has to choose her preferred option from the set of lotteries. In a perfect world without environmental restrictions, she simply chooses according to her preferences. More precisely, she chooses the option maximizing her expected utility. We refer to this case as *standard expected utility.*

Our approach extends this model by introducing the cost of distress κ, which, however, only becomes effective if the respective terminal outcome lies below the minimal subsistence level τ. For continuous probability distributions on X, the decision maker maximizes

$$V_L = \int_X u(\omega + x - h(\omega + x))dF(x)\,, \tag{C.1}$$

where h is the *cost function* depending on the terminal outcome $\omega + x$. Consistent with our assumptions, h takes the simple form

$$h(\omega + x) = \begin{cases} 0 & \text{if } \omega + x \geq \tau \\ \kappa & \text{otherwise} \end{cases}\,, \tag{C.2}$$

where $\kappa > 0$. We refer to the case where options in the choice set include the risk of falling below the minimal subsistence level as *expected utility with constraints.*

Depending on whether or not τ lies on X, one of three cases applies. First, if all achievable terminal outcomes $\omega + x \in X$ lie above the minimal subsistence level τ, the model reduces to standard expected utility. This is the case if either the minimal subsistence level τ is sufficiently small or the decision maker's assets ω are sufficiently large so that the constraint does not bind. Second, if all possible terminal outcomes lie below the minimal subsistence level τ, the whole outcome space is shifted to the left by κ. This special situation is only of limited interest for our analyses and we do not discuss it in detail. Third, and most interesting, if the minimal subsistence level τ lies within the bounds of the outcome space X, the decision maker may have an incentive to systematically depart

[21] As nonlinear probability weighting is not only found in temporal settings, an appropriate model must also predict it in settings where time plays no role (see also Epper et al. (2010)).

from standard behavior. In what follows, we will mainly focus on this particular case.

C.3.4 Illustration

Having discussed our modeling assumptions, we are now prepared to take a closer look at the mechanism generating apparent expected utility violations. We do this by studying the impact of environmental factors on risk taking behavior. For illustrative purposes, we make the following two assumptions: First, we only focus on the most important case where the minimal subsistence level lies between the smallest and the largest terminal outcome achievable by the lottery, i.e. $\tau \in (\omega + \underline{x}, \omega + \overline{x})$. Second, we restrict our attention to a binary gain lottery $L = (\overline{x}, p; \underline{x})$, where $\overline{x} > \underline{x} \geq 0$ and p denotes the probability of the larger lottery outcome to materialize.

To analyze the gap between risk preferences and risk taking behavior, we compare two utility functions: The utility function representing the decision maker's preferences (see Section C.3.1), henceforth referred to as *true utility function* u, and the utility function on the grounds of which the researcher infers risk attitudes, henceforth referred to as *ascribed utility function* $\tilde{u}$.[22] This analysis is possible since our model still retains the EU axioms on the level of preferences.

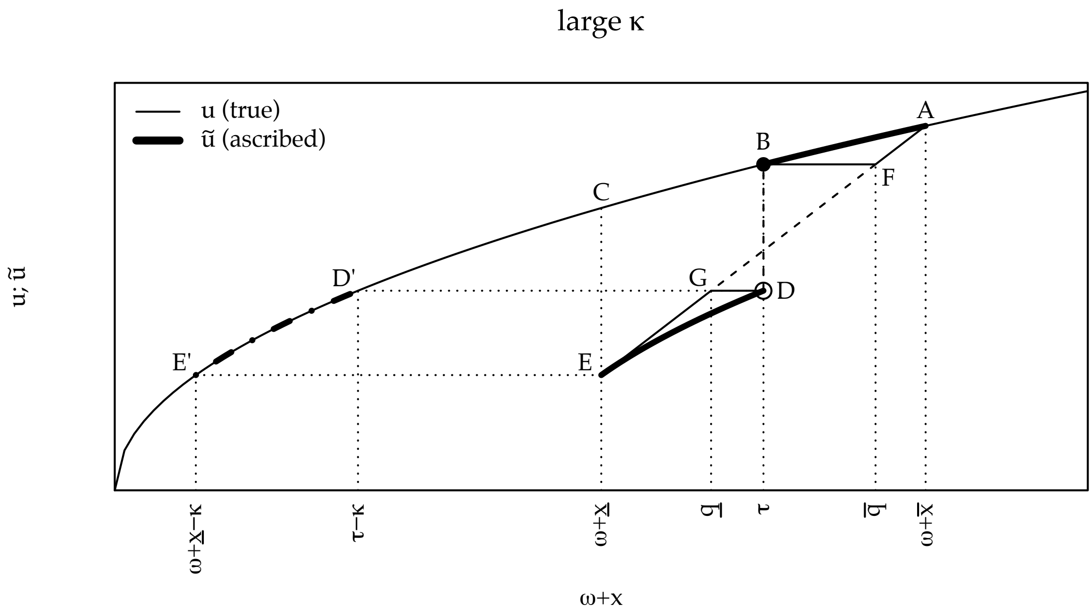
Figure C.1 plots these two utility functions. The thin, globally strictly concave curve is the decision maker's true utility function. Again, concavity of the function implies that the decision maker has risk-averse preferences. That is, in the absence of environmental constraints, the certain amount making the decision maker indifferent to the lottery L, her certainty equivalent, lies below the expected value of the lottery.[23] The line $\overline{CA}$ connecting the utility derived from the smaller terminal outcome with the utility derived from the larger terminal outcome therefore lies below u for any terminal outcome on the interval $(\omega + \underline{x}, \omega + \overline{x})$. Hence, if the decision maker faces no constraints, her risk taking behavior is entirely determined by her risk preferences.

In the presence of constraints of the form described above, however, optimal behavior may depart from true preferences. In such settings, all terminal outcomes below the minimal subsistence level are subject to costs of distress κ. The decision maker integrates these costs, and, consequently, she evaluates her true utility function at $\omega + x - \kappa$ instead of $\omega + x$ for these terminal outcomes. The relevant segments on u are then $[E', D')$ and $[B, A]$

[22] An alternative way to investigate the implications of our model is to decompose the risk premium into two parts: the contribution preferences have to risk taking behavior and the contribution the environment has to risk taking behavior. A previous version of this paper followed this route.

[23] This is the case since $u(\mathbb{E}(L)) > \mathbb{E}(u(L))$ holds (Jensen's inequality).

Figure C.1: True vs. Ascribed Utility Function I



instead of $[C, A]$. Since the costs of distress κ are not observable, the decision maker's risk attitudes are inferred from her choices over the objectively given lottery $L = (\overline{x}, p; \underline{x})$. Therefore, the ascribed utility function $\tilde{u}$ is defined as

$$\tilde{u}(z) = \begin{cases} u(z - \kappa) & \text{if } \omega + \underline{x} \leq z < \tau, \\ u(z) & \text{if } \tau \leq z \leq \omega + \overline{x}, \end{cases} \tag{C.3}$$

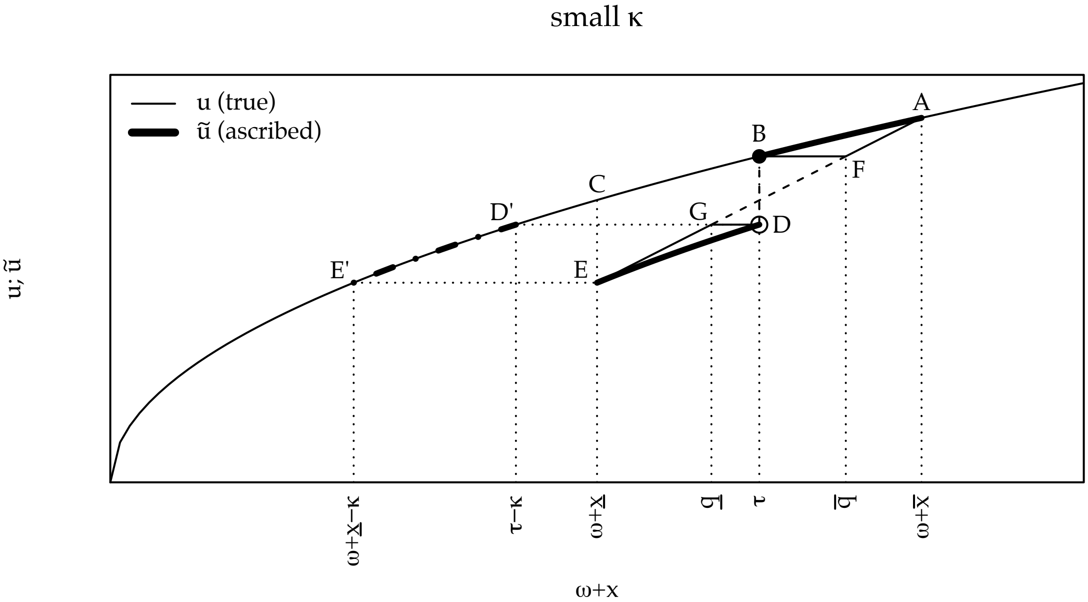
represented by the fat curves between $[E, D)$ and $[B, A]$ in Figure C.1. Due to the discontinuity of $\tilde{u}$ at τ there are four distinct regions impacting lottery valuation. Starting at the lower bound of the outcome space, $\omega + \underline{x}$, we find:

1. If $\mathbb{E}(L) \in [\omega + \underline{x}, \underline{q})$: $\mathbb{E}(\tilde{u}(L)) = p\tilde{u}(\omega + \overline{x}) + (1 - p)\tilde{u}(\omega + \underline{x})$.

   In this region, the line $\overline{EG}$ represents expected utility $\mathbb{E}(\tilde{u}(L))$. There, for $p > 0$, $\mathbb{E}(\tilde{u}(L)) < \tilde{u}(\mathbb{E}(L))$, implying risk-seeking behavior. Note that the line connecting E with A is much steeper than the respective one connecting E' and A. Hence, relative to the true unconstrained preferences, observed lottery valuation overreacts to changes in the expected value $\mathbb{E}(L)$ and, hence, to changes in p.

2. If $\mathbb{E}(L) \in [\underline{q}, \tau)$: $\mathbb{E}(\tilde{u}(L)) = u(\tau - \kappa)$.

Figure C.2: True vs. Ascribed Utility Function II



If the expected outcome exceeds the threshold $\underline{q}$ but does not meet the minimal subsistence level τ, the maximum attainable utility level is $u(\tau - \kappa)$, i.e. lottery evaluation is completely insensitive to changes in expected value. Behavior still looks risk seeking as $\mathbb{E}(\tilde{u}(L)) < \tilde{u}(\mathbb{E}(L))$.

3. If $\mathbb{E}(L) \in [\tau, \overline{q})$: $\mathbb{E}(\tilde{u}(L)) = \tilde{u}(\tau)$.

   As in the second case, lottery valuation is unresponsive to changes in expected value here. As long as expected values lie below the threshold $\overline{q}$, only subsistence utility $\tilde{u}(\tau)$ can be attained. At τ, $\mathbb{E}(L) = \tilde{u}(\mathbb{E}(L))$ holds, implying risk neutrality. At higher expected outcomes risk aversion prevails as $\mathbb{E}(\tilde{u}(L)) > \tilde{u}(\mathbb{E}(L))$.

4. If $\mathbb{E}(L) \in [\overline{q}, \omega + \overline{x}]$: $\mathbb{E}(\tilde{u}(L)) = p\tilde{u}(\omega + \overline{x}) + (1 - p)\tilde{u}(\omega + \underline{x})$

   The final case constitutes classical risk aversion, with the same feature of overreaction as in the first case.

There are three major insights from this analysis.

First, the minimal subsistence level τ demarcates the ranges over which risk seeking and risk aversion are observed, with risk neutrality at τ. Hence, increasing τ enlarges the region of risk seeking behavior and compresses the respective one of risk aversion. The interval $(\underline{q}, \overline{q})$ delimits the range over which lottery valuation is insensitive to expected value and, hence, to probability p.

Second, as changes in expected value for a given lottery range $[\underline{x}, \overline{x}]$ are driven solely by changes in probability p, our analysis implies the following pattern:

- overreaction to increasing p over $[\omega + \underline{x}, \underline{q}]$,
- insensitivity to p over $(\underline{q}, \overline{q})$,
- overreaction to increasing p over $[\overline{q}, \omega + \overline{x}]$.

We will explore this issue further in Section C.4 below.

Third, aside from the minimal subsistence level τ, behavior is driven by the costs of distress κ. Figure C.2 displays, *ceteris paribus*, the situation for a smaller magnitude of κ than is used in Figure C.1. Comparing the two figures shows that a reduction in the costs of distress changes the slope of $\overline{EA}$, moving it closer to the unconstrained line $\overline{E'A}$. This movement implies that distortions are less pronounced than for larger κ. It already follows from this findings that the minimal subsistence level induces probability distortions, the strength of which are governed by the costs of distress. The size of the interval of insensitivity, $(\underline{q}, \overline{q})$, is mainly driven by the curvature of the utility function.

Our findings so far emphasize that the environment can force the decision maker to exhibit behavior departing from her true preferences. In the next section we show what this means in particular for revealed choice behavior.

C.4 Predictions

On the following pages, we show that our model can predict a broad range of empirical findings. We devote special attention to the fourfold pattern of risk attitudes, the behavioral regularity capturing the coexistence of gambling and insurance, and demonstrate that the mechanism underlying our model can generate probability distortions, a major constituent of many novel theories of risky choice. A series of additional predictions are presented in a separate subsection. Among others, these cover common ratio violations, stake effects and the heterogeneity in risk taking behavior.

C.4.1 The Fourfold Pattern of Risk Attitudes

A robust empirical regularity documented by the empirical literature is the fourfold pattern of risk attitudes (see Tversky and Wakker (1995) for an elaborate discussion). Risk taking behavior is best characterized by (i) risk seeking for low-probability gains and high-probability losses, and (ii) risk aversion for high-probability gains and low-probability losses.

Here, we show that such behavior can naturally arise in a constrained environment, even for decision makers with expected utility preferences. We proceed as follows. In a first part, we illustrate our predictions concerning probability- and sign-dependent risk attitudes. In a second part, we give a brief comparative static analysis which elaborates on the drivers of systematic departures from standard expected utility. In a final part, we discuss how these findings motivate probability weighting, and how our predictions compare to the empirical evidence.

Our model suggests that lotteries with possible terminal outcomes below the minimal subsistence level are treated differently from lotteries without this feature. The direct cost associated with not meeting the minimum subsistence level drives a wedge between the decision maker's risk preferences and her revealed risk taking behavior. We expect that systematic departures from standard expected utility are most pronounced when environmental constraints only apply to a subset of terminal outcomes in the choice set. As already noted in the last section, it seems therefore apparent that how strong the decision maker departs from standard behavior depends on the respective weight (i.e. probability) assigned to the utility derived from the respective terminal outcome. In what follows, we show that this is indeed the case. We illustrate behavior for the most typical situation where the decision maker has to state certain amounts making her indifferent to binary lotteries with varying probabilities.

We consider two binary lotteries: a gain lottery $L_g = (\overline{x}, p; \underline{x})$ and a loss lottery $L_l = (-\overline{x}, p; -\underline{x})$, where $\overline{x} > \underline{x} > 0$. The outcomes of the two lotteries only differ in sign, but not in their absolute amount. p denotes the probability that the more extreme outcome, i.e. the larger gain or the larger loss, materializes.

We analyze risk taking behavior by means of relative risk premia. The relative risk premium rrp is the difference between the lottery's expected value $\mathbb{E}(L)$ and the (predicted) certainty equivalent $\tilde{ce}(L)$, normalized by the absolute expected value, i.e. $rrp = (\mathbb{E}(L) - \tilde{ce}(L))/|\mathbb{E}(L)|$. $rrp > 0$ indicates risk-averse behavior, $rrp < 0$ risk-seeking behavior, and $rrp = 0$ risk neutral behavior.

For a given lottery, our decision maker states her certainty equivalent $\tilde{ce}(L)$. For the case where the minimal subsistence level lies between the worst and best terminal outcome, the certainty equivalent is implicitly defined by

$$\tilde{ce}(L) = u^{(-1)}[pu(\omega + \overline{x}) + (1-p)u(\omega + \underline{x} - \kappa)] - \omega + h(\omega + \tilde{ce}(L))\,. \tag{C.4}$$

The worse terminal outcome from the lottery, $\omega + \underline{x}$, is always associated with costs of distress κ, and, hence, the utility derived from this terminal outcome is $u(\omega + \underline{x} - \kappa)$. It does, however, depend on the actual location of the terminal outcome derived from the certainty equivalent, $\omega + \tilde{ce}(L)$, of whether or not the certain option is also affected by these costs. Only if the terminal outcome from the certainty equivalent falls below the minimal subsistence level, the last term $h(\cdot)$ is equal to κ, but it is zero for all other cases. Consequently, it is required to approximate predicted certainty equivalents numerically. We do this by the algorithm described in Appendix C.6. Obviously, this procedure is not required for calculating certainty equivalents under standard expected utility. For this case, all κ's in Equation C.4 cancel out.

Figure C.3: Predicted Relative Risk Premia

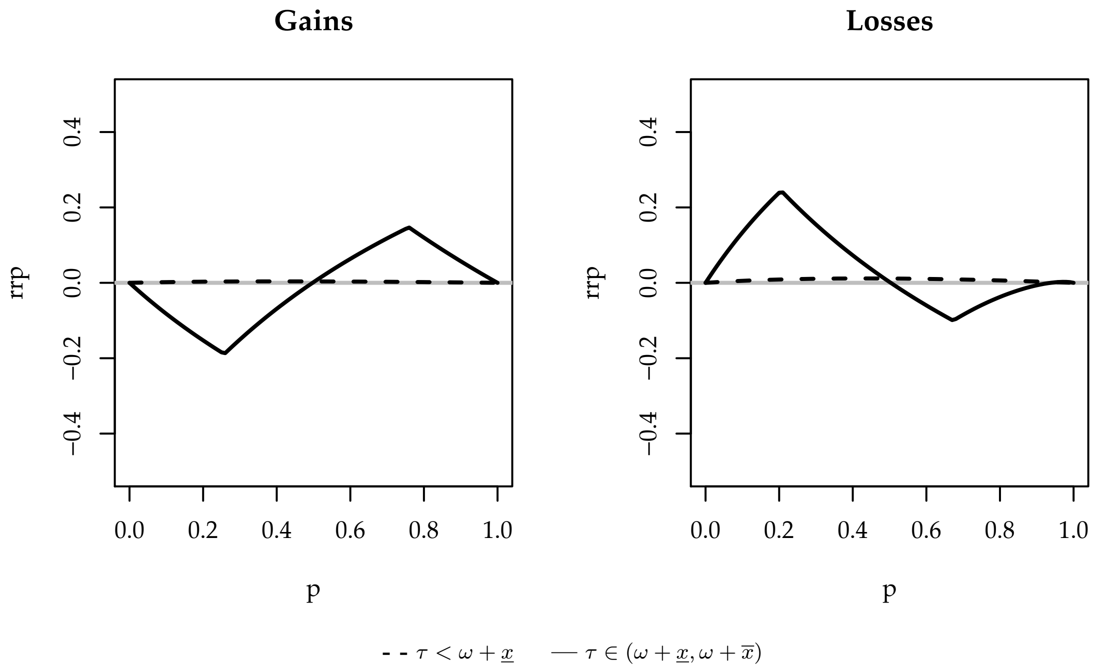


Figure C.3 depicts our findings. It plots predicted relative risk premia for both, the gain lottery $L_g = (20, p; 10)$ (left panel) and the loss lottery $L_l = (-20, p; -10)$ (right panel) for varying probabilities p and the parameter configuration presented below. It does this for two situations. First, where every attainable terminal outcome in the choice set exceeds the minimal subsistence level, i.e. where $\tau < \omega + \underline{x}$ (dashed curves), and, second, where only the better lottery outcome allows the decision maker to reach the minimal subsistence level, i.e. where $\tau \in (\omega + \underline{x}, \omega + \overline{x})$ (solid curves). The graphical illustrations are based upon the following parameter values. The utility function takes the CRRA form $u(\omega + x) = (\omega + x)^{0.8}$. We set $\omega = 30$ to ensure that all terminal outcomes are positive. The cost of distress are held fixed at $\kappa = 10$ for gains and losses, and the minimal subsistence level is either $\tau = 45$ (gains) and $\tau = 15$ (losses) for the situation where τ lies on the outcome space, or zero for the situation where τ lies below the minimal terminal outcomes in reach.

The following results arise. First, if attaining the minimal subsistence level is not of concern, the decision maker does not depart from standard expected utility. In this case, risk aversion is solely determined by her risk preferences. This result is illustrated by the dashed curves in Figure C.3. Second, if only the larger gain or the smaller loss allows the decision maker to satisfy the requirement, behavior systematically departs from this standard prediction. If τ lies between the worst and best terminal gain, the decision maker exhibits risk-seeking behavior for small-probability gains and risk-averse behavior for large-probability gains. If, instead, τ lies between the worst and best terminal loss, the decision maker exhibits risk-averse behavior for small-probability losses and risk-seeking behavior for high-probability losses.

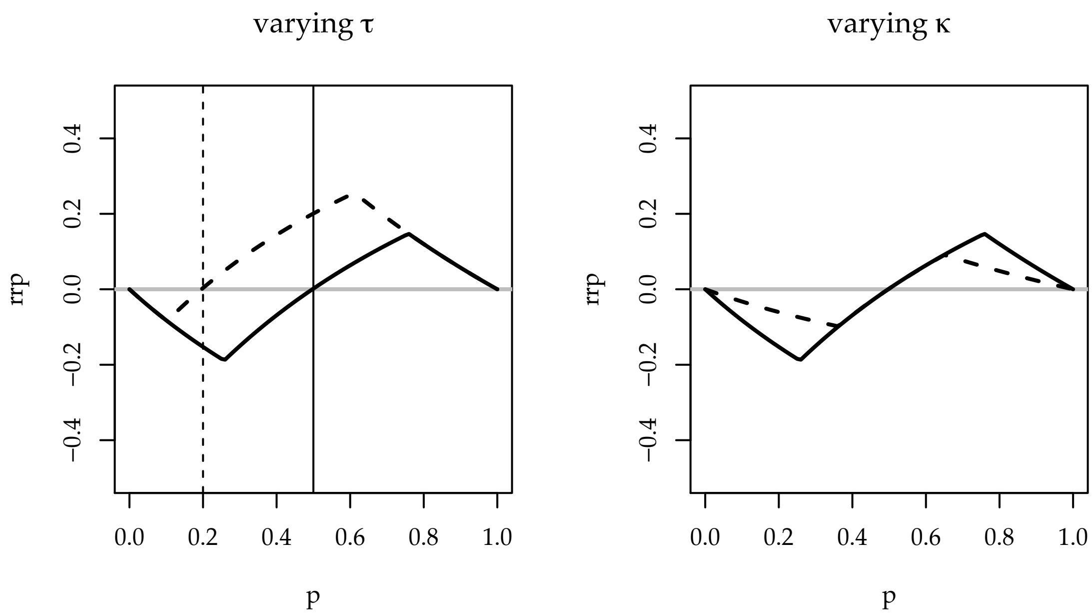
To see why these patterns emerge, reconsider the intuition provided earlier. When the probability of the better outcome is relatively small, the decision maker's terminal outcome from the certainty equivalent is more likely to lie below the minimal subsistence level. There, the pressure imposed by the environment, in our model represented by the cost of distress, forces her to opt for the more risky option. Only this choice gives her the opportunity to meet the minimal subsistence level. The opposite reasoning holds for the case where the probability of the better outcome is relatively large.

It also follows from this intuition why the *rrp*-curves take on such shapes. If it is impossible that the better (or worse) outcome materializes, i.e. if $p = 0$ ($p = 1$) for gains and $p = 1$ ($p = 0$) for losses, the decision maker's certainty equivalent collapses with the worse (better) lottery outcome. Hence, at the endpoints of the probability scale, the relative risk premium equals zero. Behavior appears more risk seeking as the better lottery outcome gains more weight (i.e. probability), but the decision maker's certainty equivalent still lies below the minimal subsistence level. The risky option becomes most attractive when the distance between the certainty equivalent and the expected value grows largest. As the probability of the better outcome further increases, it more and more compensates for the cost of distress associated with the certainty equivalent, leading the decision maker to exhibit risk neutral behavior where the two factors fully counterbalance each other. The reverse argument holds for the part of the curves where the relative risk premium is positive.

These results also point out one particular limitation of our approach. According to our model, a decision maker can never exhibit the above described systematic departures for both, gains and losses, at the same time. Depending on where the minimal subsistence level τ lies on the outcome space, either gain *or* loss behavior is affected, but never both. As we argue below, however, this finding does not necessarily contradict the empirical evidence. In fact, we still predict the fourfold pattern of risk attitudes on the aggregate level, and, under certain conditions, even on the individual level. As we do not claim that our approach is the only possible explanation for this phenomenon, however, this *feature* of our model may serve as a natural starting point to distinguish different sources of expected utility violations.

One of the central questions we have yet to answer is how the environment drives these systematic departures. A comparative static analysis provides an answer to that question. For the sake of brevity, we restrict our attention to gains.

Figure C.4: Comparative Statics for Gains



left panel: — τ =45; - - τ =42
right panel: — κ =10; - - κ =4

Figure C.4 plots relative risk premia for two different levels of minimal subsistence τ (left panel) as well as of cost of distress κ (right panel). The solid curves as well as the parameters not listed in the legend are identical to those in Figure C.3. Consistent with our findings in the previous section, it turns out that the actual location of τ on the outcome space governs for which probabilities the decision maker exhibits risk-averse and risk-seeking behavior, respectively (see left panel). The closer the minimum subsistence level lies to the worst terminal outcome (here: closer to $p = 0$), the smaller is the range of probabilities for which risk-seeking behavior is predicted, ceteris paribus. Moreover, departures are much more symmetric for the case where the minimal subsistence level splits the outcome space into two parts of equal size (solid curve) than for the case where it splits it by ratio 2:8 (dashed curve). The vertical lines in the left panel mark the actual location of τ in the probability distribution. These lines actually define the probabilities for which an expected *value* maximizer exhibits risk neutral behavior, i.e. they mark the positions where τ is exactly equal to such a decision maker's certainty equivalent, in this case $\tilde{ce} = ev$. More generally, for $\tau = \tilde{ce}$, an expected *utility* maximizer would never have an incentive to depart from standard behavior irrespective of whether she is risk averse or risk seeking.

The right panel makes clear that the extent of departures from standard expected utility is predominantly driven by the size of the cost κ the decision maker faces if the minimum subsistence level is not met. The figure plots *rrp*-curves for two configurations, a smaller cost of distress (dashed curve) and a larger cost of distress (solid curve). As the cost increases, the area under the curve with respect to the horizontal line (risk neutrality) becomes larger, indicating that the environmental force acting on the decision maker's behavior grows stronger. This finding conforms with the result reported in Section C.3.4.

The results we obtained so far provide an explanation for why models incorporating nonlinear probability weighting often describe empirical data much better than standard expected utility (Starmer, 2000). To show this, we implement the following idea: Suppose that a researcher observes stated certainty equivalents for binary lotteries generated by a hidden data-generating process (our model) and fits a RDU model (Quiggin, 1982) to the data. According to RDU, a decision maker values the lottery $L = (\overline{x}, p; \underline{x})$, with $\overline{x} > \underline{x} > 0$, by

$$\begin{aligned} u(\omega + ce(L)) &= g(p)u(\omega + \overline{x}) + (1 - g(p))u(\omega + \underline{x}) \\ &= g(p)\left[u(\omega + \overline{x}) - u(\omega + \underline{x})\right] + u(\omega + \underline{x}), \end{aligned} \tag{C.5}$$

where $ce(L)$ is the certainty equivalent and g is the probability weighting function. Rearranging this indifference condition, solving for g, and substituting $ce(L)$ by $\tilde{ce}(L)$, the certainty equivalent predicted by our model, leads to

$$\tilde{g} = \frac{u(\omega + \tilde{ce}(L)) - u(\omega + \underline{x})}{u(\omega + \overline{x}) - u(\omega + \underline{x})}\,, \tag{C.6}$$

where $\tilde{g}$ denotes the *ascribed probability weight.* Without making parametric assumptions about the weighting function, this procedure allows us to evaluate which part of observed risk attitudes the researcher would attribute to probabilistic risk aversion. Using the same model parameters and utility function as before (see Figure C.3) and inserting them into Equation C.6 further permits us to control for the contribution the decision maker's preferences and her assets make to risk taking behavior. Hence, the procedure we apply gives us an indication about how the (exogenous) environmental factors drive departures from linear probability weighting.

Figure C.5: Ascribed Probability Weights

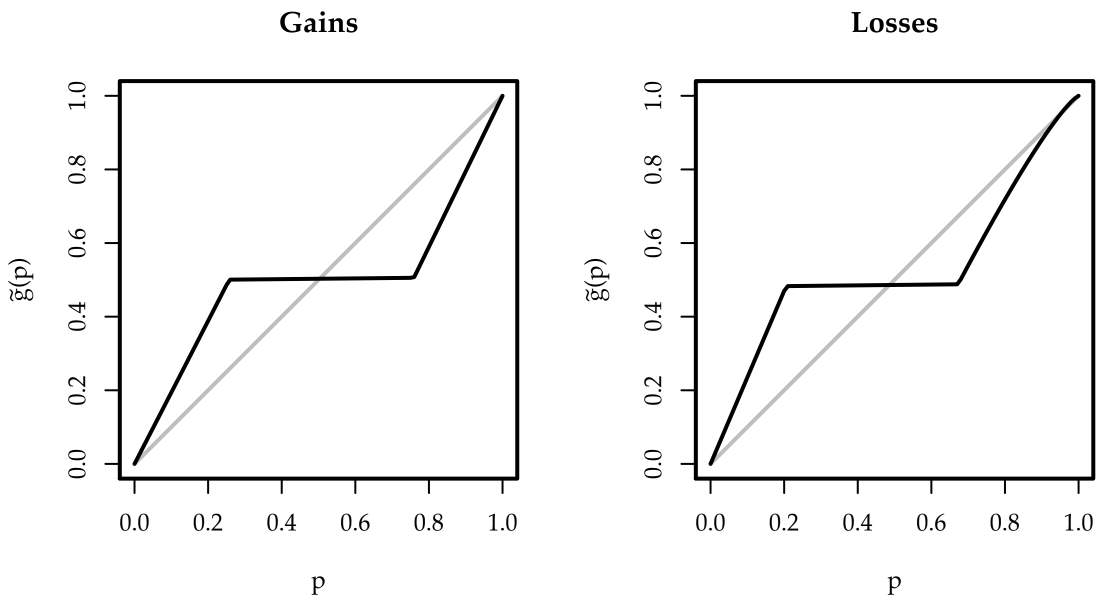


Figure C.5 shows such ascribed probability weights for gains and losses.[24] As probability weighting is just another way to account for the patterns described above, the

[24]Note that, as above, the gain and the loss weighting curves are plotted for decision makers facing different environments.

comparative static results and the interpretations we obtained earlier also apply here. While we do not recap these findings, the illustrations in Figure C.5 make a number of points clear.

First, according to our model, risk taking behavior is probability dependent.[25] If the minimal subsistence level lies within the range of possible terminal outcomes, ascribed probability weights look *as if* small probabilities were overweighted and large probabilities were underweighted (black curves). The position of the minimal subsistence level in the probability distribution governs how large the convex part of the probability transformation function is.[26] The negative consequence the decision maker faces when the minimal subsistence level is not reached largely governs how strong she departs from linear probability weighting (diagonal line). Although, in reality, the underlying data-generating process may be unobservable, this result indicates why models incorporating nonlinear probability weighting mostly provide a superior fit compared to models which do not.

Second, the illustration makes apparent how the environment can affect an expected utility maximizer's behavior. In particular, it is her striving to reach a terminal outcome equal or higher than the minimal subsistence level which rationalizes apparent probability distortions. For the situation where constraints are not binding, behavior is equivalent to standard predictions (diagonal line), but it appears to depart from linear probability weighting when the environment comes into play (black curves).

Third, it directly follows from these results that our approach can also explain the gap in the valuation of certain and uncertain outcomes found in the empirical literature (see e.g. Tversky and Kahneman (1992)). The environment can force the decision maker to favor risky options over certain options in the region where the probability of the larger gain is small, but it can force the decision maker to favor the certain option in the region where the probability of the larger gain is large. This gives rise to a certainty effect.

Fourth, the resulting picture clearly fits well to the empirical evidence at the individual level. Gonzalez and Wu (1999), for instance, provide non-parametric estimates of individual decision weights for gains. The vast majority of their subjects are insensitive to changes at medium probabilities. Their findings therefore correspond well to the flat region depicted in Figure C.5 for these probability levels.

Finally and most interestingly, while it is often argued that nonlinear probability

[25] As a consequence, our model can also explain why subjects make skew-loving choices. See Astebro et al. (2009) for an empirical study on this.

[26] If τ lies close to $\omega + \underline{x}$, the convex part of the function is maximal. The location of $\underline{q}$ and $\overline{q}$ (see last section) determine the range of probabilities for which the decision maker is insensitive.

weighting is the result of perceptional biases, motivational reasons, anticipated emotions or the result of an interplay between emotional and cognitive processes (see literature review in Section C.2), our analysis points out that such patterns may naturally arise for (rational) expected utility maximizers facing environmental constraints.

So far, we presumed that the decision maker states certainty equivalents for lotteries which vary in probabilities, but not in outcomes. While it is apparent that the same intuition we provided is also applicable for choices between two or more non-degenerate lotteries, it is important to understand what implications our model has for choices over broader sets of lotteries. This is of particular interest since most risky choice experiments confront subjects with a variety of lotteries differing in both arguments, probabilities and outcomes. Our discussion above made clear that environmental constraints may not affect all lottery choices. According to our model, a decision maker will not depart from standard expected utility if she faces the choice between lotteries with all possible terminal outcomes far beyond the minimal subsistence level. We predict, however, that this is not the case for choices between lotteries involving terminal outcomes not satisfying this requirement. This hypothesis is testable. If departures from standard predictions are less pronounced when probability distributions are shifted to the right on the outcome space, this would support our conjectures. Hershey and Schoemaker (1980) do exactly that. They find risk-seeking behavior for low and medium probability gains as well as high and medium probability losses, whereby this behavior is much more pronounced for small monetary amounts as opposed to large monetary amounts.[27] It is therefore important to bear in mind the effect the composition of lotteries may have on the prevalence of certain behavioral patterns in the data. According to our model, the structure of the data will fundamentally differ from that postulated by models assuming separability of marginal utility and probabilistic risk attitudes. This may at least partly explain why such models often do a rather bad job when it comes to predicting behavior for varying stake sizes or different situations.

A large amount of empirical evidence exists on the fourfold pattern of risk attitudes. Most studies, however, only report choice frequencies of subjects exhibiting risk-averse or risk-seeking behavior for different levels of probabilities and outcome sign (Fishburn and Kochenberger, 1979; Hershey and Schoemaker, 1980; Payne et al., 1984; Cohen et al., 1987; Wehrung, 1989; Bruhin et al., 2010).[28] Evidence on the individual level, however,

[27] More precisely, they find risk-seeking behavior for probabilities of higher gains and smaller losses between 5-44%.

[28] These results are usually complemented by statistical tests showing that more subjects exhibit certain behavior in a particular context, or that they are more risk averse in one category than another. This

is scarce. An exception is the study by Tversky and Kahneman (1992). They report for each individual the percentage of risky choices conditional on both, probability range and outcome sign, and find considerable support for the fourfold pattern. Like most evidence on the aggregate level, they only use hypothetical lottery choices, however.[29] Note that our model postulates that the rational decision maker evaluates lottery outcomes by first integrating them with her assets. It is not clear, however, whether the lack of appropriate incentives will prevent her from making such efforts.[30]

Our model predicts the fourfold pattern on the aggregate level. This is the case when subjects are sufficiently heterogenous with respect to the minimal subsistence level they face and the assets they hold. The co-occurrence of probability-dependent risk attitudes for gains and losses can then be the result of aggregation. Our model also predicts the fourfold pattern on the individual level if one of the following conditions holds.

First, for real monetary incentives we predict this pattern for common framings of losses. Since it is impossible to oblige experiment participants to pay (back) money to the experimenter, losses are often implemented using a suitable framing. Often, subjects are provided with an initial endowment from which the lottery outcomes are subtracted (see e.g. Bruhin et al. (2010)). If we take asset integration serious, a fully rational decision maker should also integrate this endowment when evaluating lotteries. She should therefore exhibit the same systematic departures for gains and for losses.

Second, the pattern can also be explained on the individual level by using an evolutionary argument. For most time during human evolution, our biological ancestors were confronted with an environment in which the essential resources such as food were scarce. The absence of markets, missing payment instruments and limited storing capacities further restricted our ancestors' scope of action. It may therefore be argued that risk attitudes are an innate characteristic of preferences which adapted over thousands of years of human evolution. In this sense, our model may provide the underlying mechanism which shaped these preferences, and, hence, may explain the Tversky and Kahneman (1992) results.

does not, however, allow to trace back the fourfold pattern of risk attitudes to the individual level.

[29]Two exceptions of studies with real monetary incentives are Cohen et al. (1987) and Bruhin et al. (2010). Both report evidence for the fourfold pattern of risk attitudes on the aggregate level, however.

[30]Many studies report differences in risk taking behavior when subjects were confronted with hypothetical as opposed to real payoffs (see e.g. Slovic (1969) and Holt and Laury (2002)).

C.4.2 Other Predictions

Common Ratio/Consequence Effects

Our model accommodates probability-dependent risk attitudes. Hence, it is no big surprise that we also predict common ratio[31] and common consequence effects, two behavioral choice patterns which (seem) to violate the independence axiom (Allais, 1953). There is one crucial difference between our approach and most previous attempts to capture these phenomena. Our approach suggests that observed violations of independence are due to exogenous constraints rather than nonlinear preferences. Although the decision maker in our model obeys the independence axiom *at the level of preferences*, she may violate it *at the level of observed behavior*.

Let us illustrate this by a simple example. We consider an expected utility maximizer with the same preferences as above, i.e. $u(\omega + x) = (30 + x)^{0.8}$. The environment restricts her from choosing according to her preferences.

Table C.1: Choice Problem

(1)	$A_1 = (\$14,\ 0.2)$	$B_1 = (\$20,\ 0.14)$
(2)	$A_2 = (\$14,\ 1)$	$B_2 = (\$20,\ 0.7)$

Table C.1 presents a typical common ratio example. It is taken from Battalio et al. (1990).[32] In this problem, the subject has to make two choices: First, between A_1 and B_1, and, second, between A_2 and B_2. The outcomes in both binary choices are the same, as are the ratios between the probabilities. A common ratio violation is observed if a subject does not consistently prefer the A- or the B-option in both cases, but either switches from A to B or from B to A. Such behavior is commonly interpreted as a violation of the independence axiom and, hence, of standard expected utility.

Battalio et al. (1990) find that about one half of their real-incentivized respondents exhibit such violations for this example. Most prefer B_1 over A_1, but A_2 over B_2.

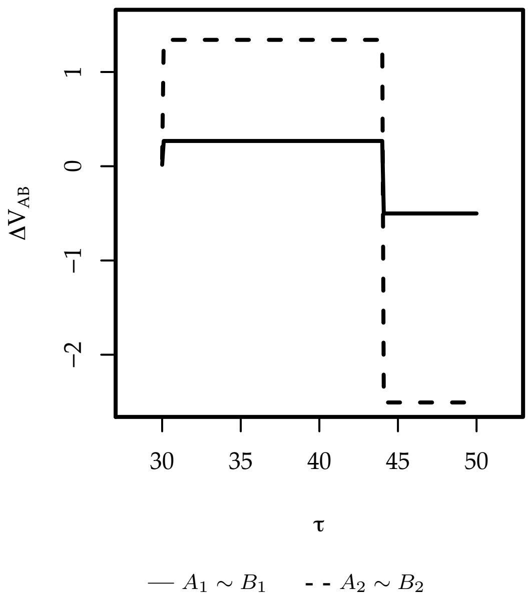
Figure C.6 demonstrates that common ratio violations are also predicted by our model. It plots the difference in valuation of the two lotteries A and B, ΔV_{AB} (ordinate), for minimal subsistence levels τ varying between the lowest and highest terminal outcome (abscissa).[33] To ensure that behavior conforms to standard expected utility, both the solid

[31] The certainty effect reported in Kahneman and Tversky (1979) and the Bergen paradox in Allais and Hagen (1979) are special cases of the common ratio effect.

[32] The choice problem is listed in Table 5 of Battalio et al. (1990) (Set 1).

[33] The vertical lines are for illustrative purposes only.

Figure C.6: Common Ratio Effect



and the dashed lines must overlap. Only in this case, a decision maker who prefers A_1 over B_1 would also prefer A_2 over B_2. This is certainly not the case here. For the regions between the horizontal lines, the decision maker switches from the A- to the B-option (for $\tau \leq \omega + x = 44$) or from the B- to the A-option (for $\tau > \omega + x = 44$), respectively. However, if τ lies far below the minimal terminal outcome in reach, our model reduces to standard expected utility. Then, the independence axiom is retained, even at the level of behavior.

Stake Effects and Heterogeneity in Risk Taking Behavior

Another prediction emerging from our model is that departures from standard expected utility are largely governed by the magnitude of outcomes, the decision maker's assets, and the minimal subsistence level. Evidently, the likelihood that the environmental constraint is binding decreases as terminal outcomes rise, i.e. as stake size and assets grow larger, and as the minimal subsistence level falls, ceteris paribus. As a result, violation of independence at the observed level should be less prevalent when stakes grow relatively large or the decision maker becomes relatively rich.[34] These departures therefore depend

[34] In the most extreme case, the cost of distress will have no noticeable impact anymore. Then, behavior will converge with the decision maker's preferences.

on the ratio between the terminal outcomes and the minimal subsistence level. The stake effect unfolds in two stages. In a first stage, the range of probabilities for which we observe risk-averse behavior will grow larger. This can be seen in Figure C.4 (solid vs. dashed curve). Relative risk aversion will therefore increase first. In a second stage, as the ratio gets even larger, standard expected utility violations become less pronounced. For high stakes, we therefore expect probability distortions to be less pronounced.

This result is particularly interesting in the light of Rabin's calibration theorem (Rabin, 2000) which states that, in order that high stake risk aversion remains within a plausible range, expected utility maximizers must be approximately risk neutral for small stakes. Our results indicate that, in the presence of constraints, this is not necessarily the case. Although the decision maker's risk preference may be constant over outcome magnitude, her revealed risk attitudes may not. The actual prediction our model makes strongly depends on the environment the decision maker is confronted with.

Our observations seems to be confirmed by recent research examining stake effects and the determinants of risk taking behavior. Fehr-Duda et al. (2010), for example, find increasing relative risk aversion for real gains when lottery outcomes are scaled up. This effect can be traced back to probability weighting and it appears that the same subjects weight probabilities less optimistically in the large stake condition. This result is qualitatively consistent with the prediction our model makes. Moreover, when taking income as a proxy measure for the likelihood that subjects face liquidity constraints, there also exists evidence supporting our conjectures. Donkers et al. (2001) find that the difference between decision weights and objective probabilities decreases in income. This result is in accordance with our prediction that richer subjects will be less prone to probability distortions.

Our model also makes predictions on the composition of risk taking types in the population. By estimating a finite mixture model, Bruhin et al. (2010) find that about 20% of their subjects do not distort probabilities, whereas about 80% do. This result is quite robust when compared across different data sets stemming from experiments conducted with college students in Switzerland and China.[35] College students typically do not have much liquidity at their disposal as they do not have a fixed monthly income. This may not only prevent (or have prevented) them from accumulating a large stock of liquid assets. As banks generally only issue credit cards to account holders if they can document guaranteed monthly earnings, they may also have problems accessing liquidity through theses channels. Our model therefore predicts the prevalence of probability distortions

[35]Conte et al. (2010) find similar results for British students.

among students. Nevertheless, we expect that the composition of behavioral types in the population will be more in favor of standard expected utility rather than RDU when richer subjects or subjects with less limited credit market access are examined.[36] Policy interventions which facilitate market access may help to counteract constraints imposed by the environment.

Choice Domain Dependency and Temporal Variation

The last point we discuss in this section addresses a finding which goes much deeper than violations of vNM axioms. Various studies report that risk taking behavior is not stable across choice domains or tasks (Weber et al. (2002); Deck et al. (2009)). MacCrimmon and Wehrung (1990), for instance, conduct a study with risk managers, and find different risk attitudes for choices involving company money and choices involving personal money. Similarly, these managers also reveal different risk taking behaviors for financial risks as opposed to recreational risks.

To be a useful concept, preferences, as a central primitive of economic behavior, should be stable across different choice domains. Today's descriptive theories of choice under risk, among them RDU theories, however, do not differentiate between revealed and true preferences. They attribute the entirety of observed behavior to preferences.[37] In contrast, our approach suggests a gap between risk preferences and risk taking behavior. It emphasizes that the environment, and in particular constraints, can play a decisive role in shaping risk taking behavior. The managers in the study by MacCrimmon and Wehrung (1990), for example, may not face liquidity constraints in their private life, but may only have limited access to financial resources on their job (e.g. spending caps).

Likewise, the empirical literature documents that, even within the same choice domain, individual risk taking behavior is subject to change over time (Baucells and Villasis, 2010; Zeisberger et al., 2010). While being relatively stable at the aggregate level, a considerable fraction of subjects change their risk attitudes if confronted with the same decision at some later point in time. Our model provides an explanation for this finding. Although preferences remain the same, the environment decision makers face may change over time. Liquidity constraints, for example, often only have a transitory character. In accordance

[36]First results of a study conducted with subject of a representative sample from the Swiss German population suggest that this is indeed the case. Using a similar design as in Bruhin et al. (2010), we find about 40% of expected utility types in our data.

[37]Clearly, these models may be a good reduced form to describe actual behavior. However, estimation results are usually interpreted as true preferences, not behavior, an interpretation which may only be appropriate in the absence of constraints.

with this conjecture, Love and Robinson (1984) find that measured risk attitudes are stable when controlling on the income level. The least risk averse farmers in their study are those in the smallest income group, whereas there is a tendency towards more risk aversion as income grows larger. In this sense, our model provides an account for both, choice domain dependency and temporal variation of risk taking behavior.

C.5 Conclusion

Our findings have material implications. In this last section, we discuss some of the most important ones.

A first point concerns rationality. Our results indicate that there are situations in which rational expected utility maximizers depart from standard predictions. In particular, we demonstrate that apparent violations of the independence axiom may be traced back to environmental constraints rather than underlying preferences. These "violations" occur since the expected utility maximizer must attain a certain minimal subsistence level imposed by the environment, but is unable to do so with the limited quantities of the commodity at her disposal and the limited market access she has. Not reaching this minimal subsistence level leads to potentially detrimental consequences. These consequences, modeled as costs, are taken into account by the decision maker when making her choice. On these grounds, our model is essentially a standard expected utility model incorporating the anticipated cost of distress.

As it turns out, it is possible to seamlessly explain a broad number of patterns found in the empirical literature which appear to be inconsistent with the standard model. Our approach, for example, can explain why gambling behavior is observed even for decision makers with risk-averse preferences. For more extreme situations, where there is no option but to take a considerable risk in order to overcome the limitations imposed by the environment, decision makers may do so as long as this risk gives them the chance to find a way out of this desperate situation. It follows from this finding that our model can generate risk attitudes depending on probabilities. Probability distortions naturally arise from decision maker's rational response to the constraints she faces. They can be rationalized without requiring violations of independence on the level of preferences. All in all, our approach closes the gap between observed risk taking behavior and true risk preferences while retaining the desirable properties of expected utility theory. It further brings out the need to account for exogenous factors driving economic behavior. In this respect, risk taking behavior may only be one single instance where apparent violations of classical economic theory can be traced back to constraints rather than to a failure of the standard preference model.[38]

The results that environmental constraints can promote probability distortions has important consequences. Probability-dependence of risk attitudes is usually ascribed to perceptional, motivational or emotional processes, or an interaction of those. The fact that fully rational decision makers may act in a similar way, however, may lead to difficulties in interpreting observed behavior. This has consequences for the design of proper policy instruments helping economic agents to behave in a more rational way without harming those that already do (Camerer et al., 2003). Such programs must be based on a sound knowledge about the actual drivers of behavior. If they are built on wrong presumptions about preferences they may be of no avail and waste money. Even worse, such policies can lead rational agents to depart from optimal behavior, and thus induce severe welfare losses. Our results stress the need for suitable mechanisms which allow to distinguish between rational and irrational sources of probability distortions. Such mechanisms can resolve the problem that a small number of observed choices are usually not informative about the real source of behavior. A natural starting point for developing such mechanisms are conditions for which competing explanations lead to distinct predictions. For example, according to our model, behavior should differ with respect to stake size and the environment the decision maker is confronted with. If economic agents depart in systematical ways from standard expected utility because they face environmental constraints, policy interventions facilitating market access may help them to behave in a way conforming with their preferences.

Our results also make another important point. Risk taking behavior depends on the choice domain, i.e. it hinges on the actual commodity under consideration. For different commodities, such as private money, company money or health, the constraints the decision maker faces are unlikely to be the same. More precisely, she may be endowed with different quantities of the respective commodity, may have different minimal subsistence levels to reach, and may face differently harmful consequences when this goal is not achieved. Moreover, as constraints are often only of a transitory character, revealed risk attitudes are likely to change over time. Models that do not take into account these disparities between domains and the dependence on a changing environment have only limited predictive power. For example, it is highly questionable whether such models together with estimated parameters from one experiment can be used to predict behavior

[38] A similar explanation can motivate systematic departures from constant time discounting (see Epper (2009)).

for other domains, other subjects or other points in time. RDU models may still be a good reduced form for describing behavior, but they are likely to be not suited well for these purposes.

These models are further challenged by the fact that potential constraints driving standard expected utility violations do not necessarily affect all choices. As long as the minimal subsistence level lies within the interval of possible outcomes, the systematic departures described above should emerge. A typical example are risky choice experiments. These experiments are often conducted among college students, i.e. subjects who are exceptionally exposed to liquidity constraints. They confront subjects with a broad number of choices over options with similar outcomes. If the constraint is binding, we would predict all anomalously appearing patterns of risk taking behavior. Systematic departures, however, may be less prevalent if choices over a wider range of outcomes are made. Nonetheless, it can be argued that the repeated struggle for survival our ancestors were confronted with may have shaped preferences in a way conforming to the typical findings. Our model may therefore provide an evolutionary foundation for the fourfold pattern of risk attitudes and a broad number of other stylized facts in choice under risk, and, hence, may also predict apparent standard expected utility violations in the absence of constraints.

It is obvious that the constraints limiting the decision maker's scope of action must not necessarily be exogenous, but may well be self imposed. The minimal subsistence level in our model may then be interpreted as an aspiration level, and the cost of not reaching the self-defined goal as psychological (or emotional) costs. While it is arguably much harder to rationalize departures from standard expected utility in this case, and such an approach is hardly testable, the predictions our model makes remain the same as in the case with exogenous constraints. In particular, probability-dependent risk attitudes still result endogenously from our model, without making additional assumptions about how probabilities are weighted.

Our findings indicate that the descriptive inadequacies of expected utility theory can, at least partly, be resolved when environmental constraints are taken into account. If the researcher would observe the decision maker's environment, she would be able to fully reproduce her real preferences - preferences which do not necessarily violate independence. Rabin and Thaler (2001) use a dead horse as a metaphor for expected utility theory. We believe that the reports of the horse's death are greatly exaggerated. While arguably being injured - there are still issues such as framing effects or the preference reversal phenomena which are barely explainable by our approach - we are confident that the horse is still nickering.

C.6 Appendix: Calculation of Certainty Equivalents

According to our model, the certainty equivalent for a given lottery is only implicitly defined. We therefore need to approximate it numerically. The following lines contain the receipt describing each step of the algorithm we implemented to do that.

1. Calculate the value of the lottery L, V_L, according to Equation C.1.

2. Split the outcome space $[\underline{x}, \overline{x}]$ into n equally spaced partitions, where n is a large number (we took 200). Note that $\tilde{ce}(L)$ lies by definition between the lowest and the highest lottery outcome.

3. Calculate the value for each of the $n+1$ amounts, $V_{\tilde{ce}(L)}$, again following Equation C.1.

4. Find the smallest and the largest amount x_l and x_h lying below and above indifference $V_L = V_{\tilde{ce}(L)}$ in the vector $V'_{\tilde{ce}(L)}$.

5. Determine the weight w representing relative distance from indifference to the x_l, i.e. $w = x_l/(x_h - x_l)$.

6. Obtain the approximate certainty equivalent $\tilde{ce}(L)$ by the weighted mean $\tilde{ce}(L) = wx_h + (1-w)x_l$.

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Curriculum Vitæ

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2007–2009	Postgraduate Studies in Applied Statistics, ETH Zurich, Switzerland.
2006	Mannheim Empirical Research Summer School, University of Mannheim, Germany.
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2000–2005	Studies in Management and Economics, University of Zurich, Switzerland. Graduation: lic. oec. publ.
1995–1999	High School (Kantonsschule), Sargans SG, Switzerland. Graduation: Matura Type E.

Publications

06/2011	"Viewing the Future through a Warped Lens: Why Uncertainty Generates Hyperbolic Discounting,", *Journal of Risk and Uncertainty*, forthcoming (with A. Bruhin and H. Fehr-Duda).
10/2010	"Risk and Rationality: The Effects of Mood and Decision Rules on Probability Weighting," *Journal of Economic Behavior and Organization*, forthcoming (with A. Bruhin, H. Fehr-Duda and R. Schubert)
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08/2007	"Rationality on the Rise: Why Relative Risk Aversion Increases with Stake Size," *SOI Working Paper*, No. 0708, University of Zurich (with A. Bruhin, H. Fehr-Duda and R. Schubert).
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